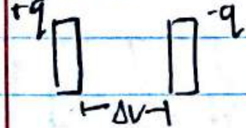
* Up to 30.6.
Ch. 32 is about Maxwell's Eqns

12/3/15

Circuits w/ Inductors

Energy stored in $\vec{E}$ field

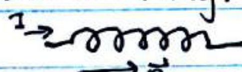


$\Delta V \propto q$ — energy stored $= \frac{1}{2}CV^2$.

$C = \frac{q}{\Delta V} \rightarrow \Delta V \sim \frac{1}{C}!$

Energy stored in magnetic field $\vec{B}$

<u>solenoid</u>



How do we find ΔV? ←

$\frac{dI}{dt} \neq 0 \rightarrow \frac{dB}{dt} \neq 0 \rightarrow \frac{d\Phi_B}{dt} \neq 0$. ← induced EMF.

so $\mathcal{E}_L \propto -\frac{dI}{dt} L$ — self-inductance (L) is constant of proportionality.

↑ opposes change.

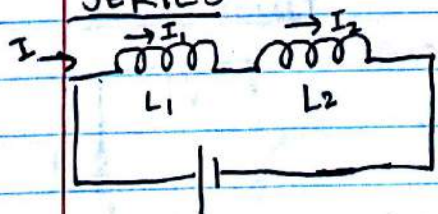
Energy stored (U_B):

$P = \frac{dW}{dt}$, $dW = P\,dt$.

$P = I\mathcal{E}_L = IL\frac{dI}{dt}$, $\int_0^t P\,dt = \int_0^I IL\,dI$.

$\therefore \boxed{W = \frac{1}{2}LI^2 = U_B}$ $(I=0,\ U_B=0)$.

<u>SERIES</u>

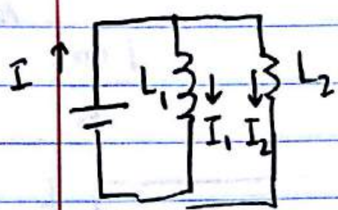


$L_1\frac{dI}{dt} + L_2\frac{dI}{dt} = V$ ← $L_{eff}\frac{dI}{dt}$

$\therefore \boxed{L_{eff} = L_1 + L_2}$

$I = I_1 = I_2$.

Parallel



$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt} = L_{eff} \frac{dI}{dt}$$

and $$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}.$$

$$L_1 \frac{dI_1}{dt} = L_2 \frac{dI_2}{dt}$$

$$\therefore \frac{dI_1}{dt} = \frac{L_2}{L_1} \frac{dI_2}{dt}$$

$$L_2 \frac{dI_2}{dt} = L_{eff}\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right),$$

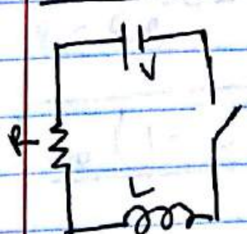
$$L_2 \cancel{\frac{dI_2}{dt}} = L_{eff}\left(\frac{L_2}{L_1} \cancel{\frac{dI_2}{dt}} + \cancel{\frac{dI_2}{dt}}\right)$$

$$L_2 = L_{eff}\left(\frac{L_2 + L_1}{L_1}\right)$$

$$L_{eff} = \frac{L_1 L_2}{L_2 + L_1}, \quad \boxed{\frac{1}{L_{eff}} = \frac{1}{L_1} + \frac{1}{L_2}}.$$

	Resistor	Capacitor	Inductor
Main eq	$\Delta V = IR$	$\Delta V = q/C$	$\mathcal{E}_L = -L \frac{dI}{dt}$
Series	$R_{eq} = \sum_i R_i$	$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$	$I_{eq} = \sum_i L_i$
Parallel	$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$	$C_{eq} = \sum_i C_i$	$I_{eq} = \sum_i \frac{1}{I_i}$

RL Circuit



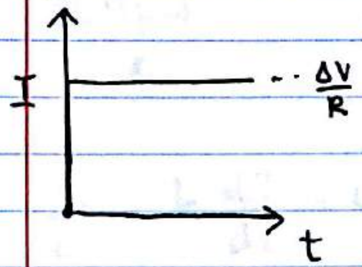
- inductors resist sudden changes by generating an opposing current.

t=0 switch open
t>0 switch closed $\Rightarrow \frac{dI}{dt} \neq 0.$

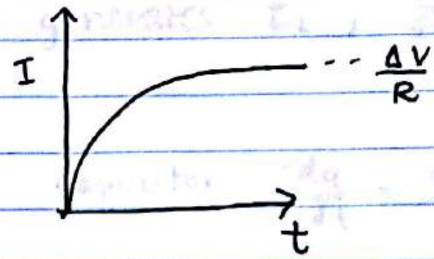
t=0 switch ~~closed~~ open
t>0 closed

RL Circuit [Cont.]

no L



with L

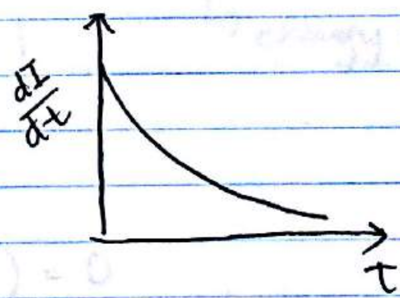


$$\mathcal{E} - IR - L\frac{dI}{dt} = 0 \rightarrow \mathcal{E} - IR = L\frac{dI}{dt}$$

$$\int_0^t \frac{dt}{L} = \int_{I=0}^{I} \frac{dI}{\mathcal{E} - IR}$$

$$\frac{1}{L}t = -\frac{1}{R}\ln(\mathcal{E} - IR)\Big|_0^I$$

$$t = \frac{-L}{R}\ln\left(\frac{\mathcal{E} - IR}{\mathcal{E}}\right)$$



so $\mathcal{E}$ becomes negligible

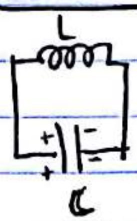
$$e^{-tR/L} = \frac{\mathcal{E} - IR}{\mathcal{E}}$$

$$\mathcal{E}e^{-tR/L} = \mathcal{E} - IR$$

$$\mathcal{E} - \mathcal{E}e^{-tR/L} = IR$$

$$\boxed{I_0\left(1 - e^{-\frac{tR}{L}}\right) = I}$$

LC Circuit



$t=0$ close switch

- capacitors starts to discharge $\rightarrow I$
- inductor generates $\mathcal{E}_L$, $\frac{dI}{dt} \neq 0$ b/c capacitor.

$$\frac{q}{C} - L\frac{dI}{dt} = 0.$$

capacitor: $\frac{-dq}{dt} = I$

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \leftarrow \text{simple harmonic motion}$$

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0.$ $\quad x(t) = x_0\cos(\omega t + \varphi)$

$\omega = \sqrt{\frac{k}{m}}$

$t=0 \rightarrow$ energy of $\vec{E}$

$\hookrightarrow$ energy of $\vec{B}$.

$$\frac{q}{CL} + \frac{d^2q}{dt^2} = 0$$

$$q(t) = q_0\cos(\omega t + \varphi)$$

$$\omega = \sqrt{\frac{1}{CL}}.$$

Plugging in,

$$\cos(\omega t + \varphi)\left(-\omega^2 + \frac{1}{LC}\right) = 0$$

for all t, valid if $-\omega^2 + \frac{1}{LC} = 0.$ // $\omega = \sqrt{\frac{1}{LC}}.$