1/19/15

Faraday's law
- Induced electromotive voltage from changing magnetic flux

Work done by magnetic field is 0 (non-conservative field)
but... change in magnetic field!

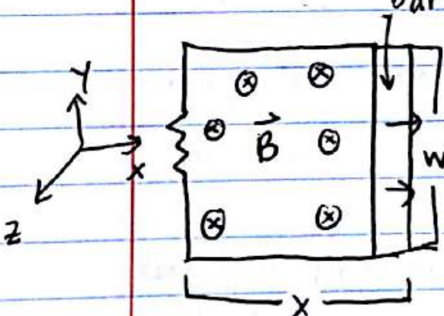
$$\mathcal{E} \propto \frac{d\Phi_B}{dt}$$

$\Phi_B = \iint_S \vec{B} \cdot d\vec{A}$] open surface! because $\oint \vec{B} \cdot d\vec{A} = 0$.

can also be function of time: $\Phi_B(t) = \iint_S \vec{B}(t) \cdot d\vec{A}(t)$

3 ways for $\frac{d\Phi_B}{dt} \neq 0$:
① Change $B(t)$
② Change $\vec{A}(t)$
③ Change $\vec{B} \cdot \vec{A}$ in time $(\theta(t))$

bar @ $v = v\hat{x}$



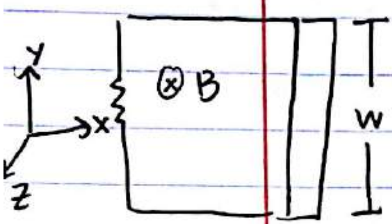
$\vec{B} = -\vec{B}\,\hat{z}$
$\vec{A} = \vec{w} \times \vec{x} \quad \hat{z}$
$\vec{v} = v\,\hat{x}$

$\vec{F}_B = q\vec{v} \times \vec{B}$
$= qv\hat{x} \times (-B)\hat{z}$
$= qvB\hat{y}$

This creates an E-field.
$\vec{F}_E = q\vec{E}$
$= qE\cdot\hat{y}$

Stable config:
$\vec{F}_B = \vec{F}_E$
$\cancel{q}E = \cancel{q}vB$

$\boxed{E = vB}$



Force on a current-carrying wire: $\vec{F} = I\vec{\ell} \times \vec{B}$ ← we don't know yet.

$= I w (\pm \hat{y}) \times B(-\hat{z})$

$= \mp I w B \hat{x}$

$\vec{F}$ cannot be positive! Or else our bar magnet would inifinitely accelerate. Our current is along the positive y-axis. (so F_B is directed left).

This induced I generates a magnetic field that opposes B.

Think of Newton's Third Law! This negative sign is Lenz's Law: a system will gen. a current to oppose the change in flux.

$\mathcal{E} = \underbrace{\int \vec{E} \cdot d\vec{s}}_{\text{integrating along the bar.}} = vBw$

This works!

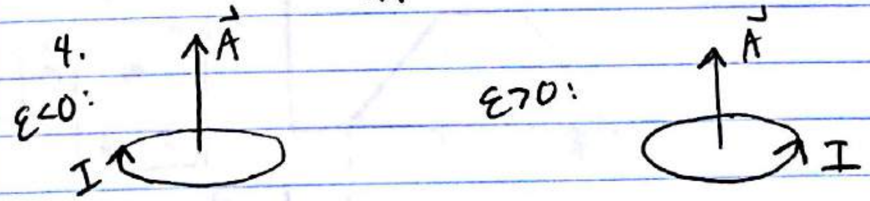
$\Phi_B = -Bwx$

$\frac{d\Phi_B}{dt} = -Bw\frac{dx}{dt}$

$= -Bwv$

Recipe for determining $\mathcal{E}$ and I:

1. Define $\vec{A}$ & commit!
2. Calculate $\Phi_B = \int \vec{B} \cdot d\vec{A}$ (usually $\vec{B} \cdot \vec{A}$)
3. Calculate $\frac{d\Phi_B}{dt} = -\mathcal{E}$
4.



$B = -B(t)\hat{z}$

$A = -\pi r^2 \hat{z}$

$$\Phi_B = B(t)\,\pi r^2$$

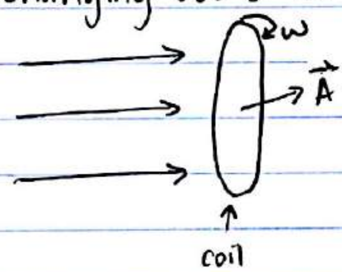
$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}, \quad \mathcal{E} = -\pi r^2 \frac{dB}{dt}$$

What if $A = \pi r^2 \hat{z}$?, $\mathcal{E} = \pi r^2 \frac{dB}{dt}$, but current is still counterclockwise (signage of $\mathcal{E}$ is w/ respect to A).

Also, $\oint_{loop} E \cdot ds \neq 0$! Only static $\vec{E}$-fields are conservative. induced $\vec{E}$-fields are not.

Changing $\cos\theta$ of $\vec{B}\cdot\vec{A}$

$B = B\hat{x}$

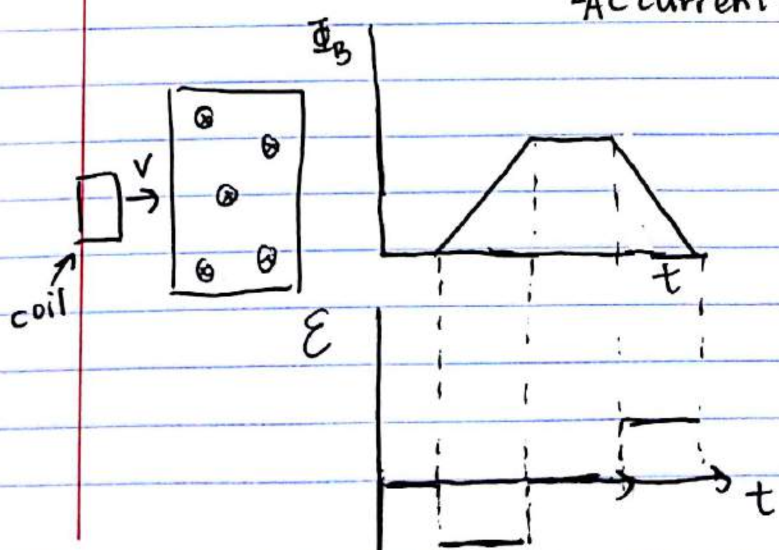


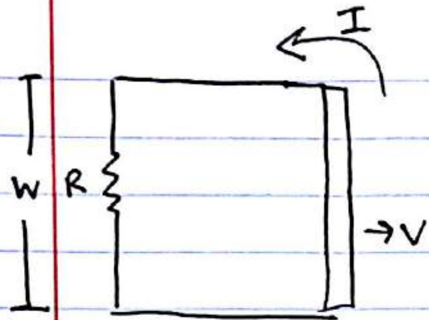
$|\vec{A}| = \pi r^2$. $\quad$ ω → rotational velocity

$$\Phi_B = BA\cos\theta$$
$$= B\pi r^2 \cos\theta$$
$$\frac{d\Phi_B}{dt} = -B\pi r^2 \sin\theta \frac{d\theta}{dt}$$

$$\mathcal{E} = \omega B \pi r^2 \sin(\omega t)$$

AC current! This is a generator.





$$\mathcal{E} = \int \vec{F}_B \cdot d\vec{s} = Bwv$$

$$I = \frac{V}{R} = \frac{Bwv}{R}$$

$$P = I^2 R = \frac{B^2 w^2 v^2}{R} \checkmark$$

$$P = Fv$$

$$= I(\vec{l} \times \vec{B})v$$

$$= \frac{Bwv}{R} wBv$$

$$= \frac{B^2 w^2 v^2}{R} \checkmark$$