11/10/15

Generating Magnetic Fields

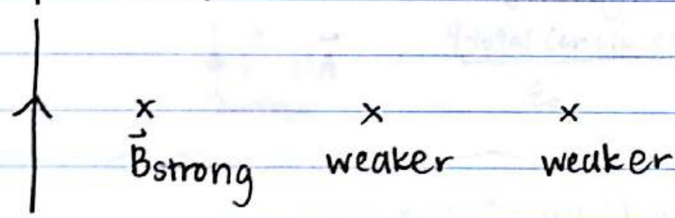
Current → generates $\vec{B}$

-We want to find direction & magnitude of this field.

Convention (much like we did with $\vec{E}$ field lines)
direction of field lines are determined by right hand rule.

By experiments, we know these generated field lines are closed-loop around the source of field.

By experiment,



there is a dependence on distance ($\vec{B}(r)$) from source
$\boxed{\frac{1}{r}}$ dependence! different from E-field's $1/r^2$ dependence.

$\vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}$, μ_0 = permeability of free space
$= 4\pi e^{-7}$ Tm/A.

I_2 feels B_1. ⇒ F_2.

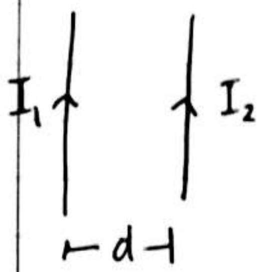
$\vec{F}_2 = I_2 \vec{L}_2 \times \vec{B}_1$

By right hand rule, $\vec{F}_2$ is attractive! ($\vec{B}_1$ into page)

(If I_1, I_2 are opposite, $\vec{F}$ is repulsisive.)

Analogy: $\vec{E}$ q_1 q_2

F attract if q_1, q_2 opp. sign
F repels if q_1, q_2 same sign



$$\vec{B}_1 = \frac{\mu_0}{2\pi} \cdot \frac{I_1}{d}$$

$$\vec{F}_2 = I_2(\vec{L}_2 \times \vec{B}_1)$$

$$= I_2 L_2 B_1 = \boxed{I_2 L_2 \frac{\mu_0 I_1}{2\pi d}}$$

$$B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{d}, \quad \vec{F}_1 = I_1(\vec{L}_1 \times \vec{B}_2)$$

$$= \boxed{I_1 L_1 \frac{\mu_0 I_2}{2\pi d}}$$

$\vec{F}_1$, $\vec{F}_2$ both dependent on L.

<u>Ampere's Law</u> ⇒ an effective way to determine $\vec{B}$.

<u>Reminder</u>: Gauss Law: count field lines to get field strength ($\vec{E}$-field).

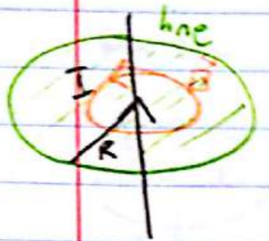
$$\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{total\,(enclosed)}}{\varepsilon_0}$$

Ampere: let's "count" lines like Gauss did! to find the <u>strength of the field</u>.

$\oint_{surface} \vec{B} \cdot d\vec{A} = 0!$ So we use: $\boxed{\oint_{line} \vec{B} \cdot d\vec{l} = I\mu_0}$ ← Ampere's Law

because $\vec{B}$ is a closed loop.

So... $\boxed{\oint_{line} \vec{E} \cdot d\vec{l} = ? = 0}$ In terms of potential, by Kirchoff's Loop Law, $\Delta V = 0$. Thus, $E = 0$ b/c $E = \frac{\partial V}{\partial x}$.



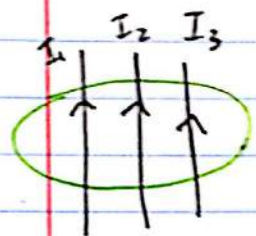
$$\oint_{line} \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B \oint_{line} d\vec{l} = \mu_0 I$$

$$\underbrace{\oint_{line} d\vec{l}}_{2\pi r}$$

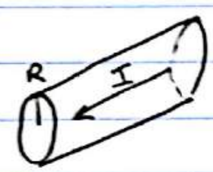
$$B 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$



$$\oint_{line} \vec{B} \cdot d\vec{l} = \mu_0 \sum_i I_i$$

$\vec{B}$ field inside & outside wire



$r > R$

$$B \oint d\vec{l} = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$r < R$

Not whole I is enclosed.

$$\oint_{line} \vec{B} \cdot d\vec{l} = \mu_0 I'$$

$$I' = I \left(\frac{\pi r^2}{\pi R^2} \right)$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \frac{r^2}{R^2}$$

$$B \oint d\vec{l} = \frac{\mu_0 I r^2}{R^2}$$

$$B 2\pi r = \frac{\mu_0 I r^2}{R^2}$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

$r = R$, $\boxed{B = \frac{\mu_0 I}{2\pi R}}$