10/13/15

F_c is a conservative force, so $-\int_A^B \vec{F}_c \cdot d\vec{l}$ is a state variable.

Determine V from E

$\Delta V = -\int \vec{E} d\vec{l}$ max $\cos\theta = 1$

Determine $\vec{E}$ from V

$dV = -\vec{E} \cdot d\vec{l}$ - Assume 1D: $dV = -E dx$

$\boxed{E = -\frac{dV}{dx}}$

1) $\vec{E}$ is slope of V in each point

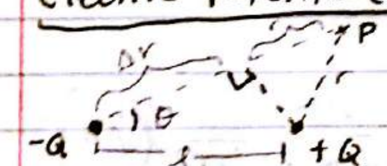
2) $\vec{E}$ is a measure of the RATE of change V

Work → Joule / work → electronvolt eV

q → e ← use this unit!

1eV is energy acquired by q = e when q is moving through a potential difference of 1 V.

Electric Potential of a Dipole



$V_p = ?$

if P is very far away, $(r >> l)$, these assumptions apply.

$V_{TOT} = \sum_i V_i$, $V_p = \underbrace{\frac{-Q}{4\pi\varepsilon_0 (r+\Delta r)}}_{V_{-Q}} + \underbrace{\frac{Q}{4\pi\varepsilon_0 r}}_{V_{+Q}}$

$= \frac{-Qr + Q(r+\Delta r)}{4\pi\varepsilon_0 r(r+\Delta r)} = \frac{Q\Delta r}{4\pi\varepsilon_0 r(r+\Delta r)}$ $l\cos\theta$

$= \frac{\overbrace{Ql}^{p}\cos\theta}{4\pi\varepsilon_0 r(r+\Delta r)}$ neglect this $(r >> l)$

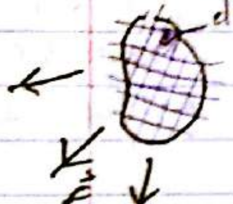
$= \boxed{\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$

V_{DIPOLE} vs. $V_{POINT CHARGE}$

$= \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ $\qquad$ $= \dfrac{q}{4\pi\varepsilon_0 r}$

→ $\frac{1}{r^2}$ goes to 0 faster than $\frac{1}{r}$. This is because far away, a dipole looks like a 0 charge.

<u>V_p of Continuous Dist. of Charge</u>



Two approaches:

1) $\sum_i V_i \rightarrow V_T = \int dV = \int \frac{dq}{4\pi\varepsilon_0 r}$

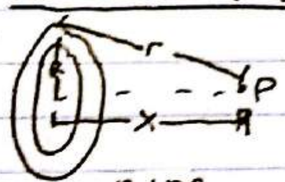
-use when charge distribution is known.

2) when $\vec{E}$ is known:

$\Delta V = -\int \vec{E} \cdot d\vec{l}$

<u>Calculate V for a ring of charge Q (approach #1)</u>

Q



$V_p = \int \frac{dQ}{4\pi\varepsilon_0 r}$ $\qquad r = \sqrt{x^2+R^2}$

$dQ = R\,d\theta\,\lambda$, $\quad \lambda = \frac{Q}{2\pi R}$

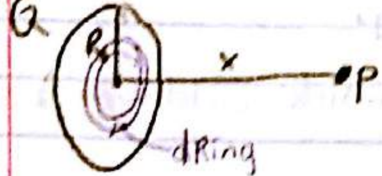
$= \frac{R\lambda}{4\pi\varepsilon_0 r}\int_0^{2\pi} d\theta = \frac{\lambda R}{4\pi\varepsilon_0\sqrt{x^2+R^2}}(2\pi)$

$= \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$

if $x \gg R$, $\boxed{V_p = \frac{Q}{4\pi\varepsilon_0 x}}$

(point chg.)

Calculate V for disk of charge Q



$$V_p = \int dV = \int \frac{dQ}{4\pi\varepsilon_0 r}$$

dQ = Q in dRing

$$\frac{\text{charge ring}}{\text{charge disk}} = \frac{\text{area ring}}{\text{area disk}} = \frac{dQ}{Q}$$

$$dQ = \frac{Q\cdot\text{area ring}}{\text{area disk}} = \frac{Q\,2\pi R\,dR}{\pi R_0^2} = Q\,\frac{2R\,dR}{R_0^2}$$

$$V_p = \int_0^R \frac{Q\,2R\,dR}{4\pi\varepsilon_0 R_0^2 (R^2+x^2)^{1/2}}$$

$$= \frac{Q2}{4\pi\varepsilon_0 R_0^2}\int_0^{R_0} \frac{R\,dR}{(R^2+x^2)^{1/2}} \qquad \left.\tfrac{1}{2}R_0^2\right|_0^{R_0}$$

$$= \frac{Q}{2\pi\varepsilon_0 R_0^2}\left[(x^2+R_0^2)^{1/2} - x\right]$$

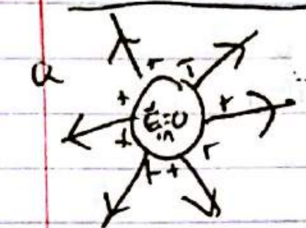
$x >> R_0$: $V_p \to$ pt. charge

$$= \frac{Qx}{2\pi\varepsilon_0 R_0^2}\left[\left(1+\frac{R_0^2}{x}\right)^{1/2} - 1\right] \quad \text{apply Taylor series}$$

$$= \frac{Qx}{2\pi\varepsilon_0 R_0^2}\left[\cancel{1} + \frac{R_0^2}{2x^2} \cancel{-1}\right]$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0 x}}$$

Electric Potential Charged Conductor (approach #2)



$d\ell = dr$ (moving radially)

$\vec{E}_p = \frac{Q}{4\pi\varepsilon_0 r^2}$ (outside conductor)

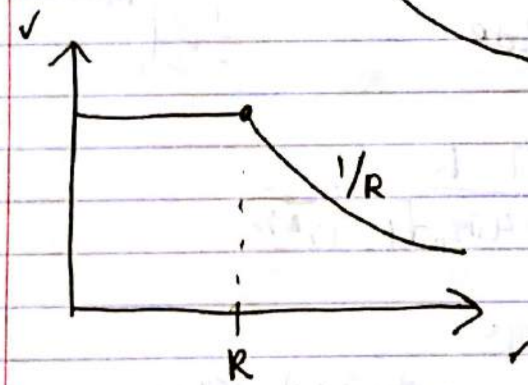
$\Delta V_{A\to B} = ?$ $\quad \Delta V_{AB} = -\int_A^B \vec{E}\cdot d\vec{\ell} = -\int_A^B E\,dr$

2 cases:

$r \geq R$ $\quad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$ $\quad -\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_A^B \frac{1}{r^2}\,dr$

$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

$r < R$ $\quad \vec{E} = 0$



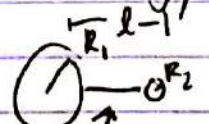
$\vec{E} = 0$, $\frac{dV}{dx} = 0$ but $V \neq 0$!
It's just constant! (equipotential) surface

So $\frac{Q}{4\pi\varepsilon_0 R^2}$ is V_p of conductor's inside.

What about a strange shaped conductor?

surface $\Delta V = 0$

Analogy: assume $\ell >> R_2, R_1$



$V_1 = V_2$

$\frac{Q_1}{4\pi\varepsilon_0 R_1} = \frac{Q_2}{4\pi\varepsilon_0 R_2}$, $\quad \frac{Q_1}{R_1} = \frac{Q_2}{R_2}$

recall $\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$

(cont.) $\frac{E_1}{E_2} = \frac{Q_1}{Q_2} \cdot \frac{R_2^2}{R_1^2} = \frac{R_2}{R_1}$ ← most intense @ top of tip