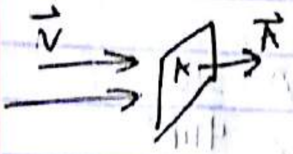
10/06/15

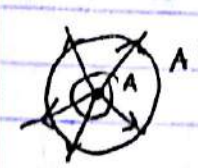
Review : Flux



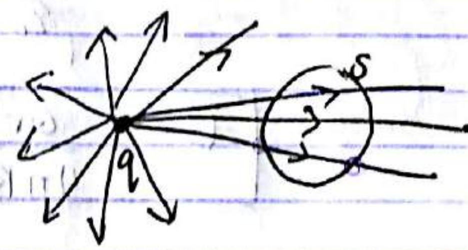
#lines through $A = \Phi$

$\Phi = \vec{E} A$, but there is a dependence on θ (between E & A)

↳ $\vec{E} \cdot \vec{A} = \Phi$



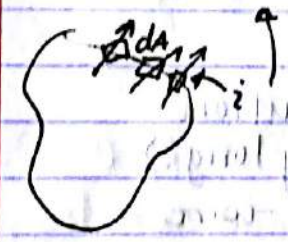
$\Phi_A = \Phi_{A'}$



$\Phi_S = 0$.

$\Phi_E = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow \oint E \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0}$ ← uniform charge distribution

$\Phi_{tot} = \sum_i \Phi_i = \oint \vec{E} \cdot dA$



Non uniform charge distribution

$\Phi_E = \oint E \cdot dA = \frac{1}{\varepsilon_0} \int_0^V \rho \, dV$ ← charge distribution

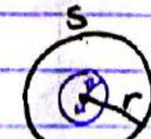
spherical symmetric distribution of charge

insulating sphere Q

Calculate E-field for $\begin{cases} r \geq R \\ r < R \end{cases}$

$r \geq R$

① Gaussian Surface →



$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

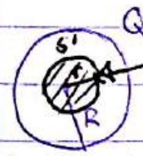
$E \oint dA = \frac{Q}{\varepsilon_0}$

$E(4\pi r^2) = Q/\varepsilon_0$

$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$

like a point charge

$\underline{r < R}$



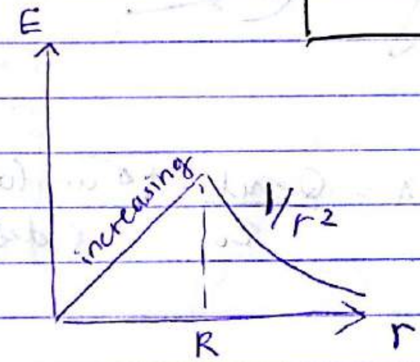
Gaussian surface

$$\Phi_E = \underbrace{\oint_{s'} \vec{E} \cdot dA}_{} = \frac{Q_{enc}}{\varepsilon_0} \qquad \rho = \frac{3Q}{4\pi R^3}$$

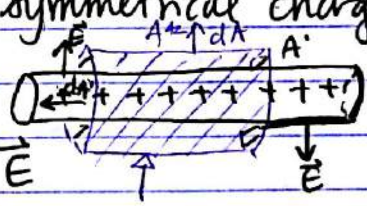
//// : Qenc

$$\vec{E}\, 4\pi r^2 = \frac{\rho V_{s'}}{\varepsilon_0} \longrightarrow \frac{3Q}{4\pi R^3} \cdot \frac{4}{3}\pi r^3$$

$$\boxed{\vec{E} = \frac{Qr}{4\pi R^3 \varepsilon_0}}$$



Cylindrical symmetrical charge distribution
(very long!)
-wire



$\underline{\text{calculate } \vec{E}}$ $\quad Q = \lambda l$

Gaussian surface $\quad$ λ: charge density per unit length

$$\Phi_E = \Phi_A + \Phi_{A'}$$

A'·E is 0!

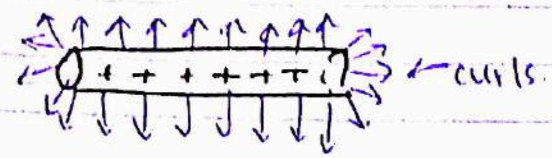
$$= \oint_A \vec{E} \cdot d\vec{A} + \cancel{\oint_{A'} \vec{E} \cdot d\vec{A}}^{\,0} \qquad \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$= \oint_A \vec{E} \cdot d\vec{A} = \vec{E} \int_A d\vec{A} = \underline{E\, 2\pi r l}$$

$$\vec{E}\, 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\varepsilon_0}, \qquad \boxed{\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}}$$

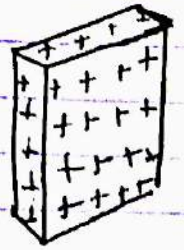
1/r relationship.

What if the wire is not infinite?



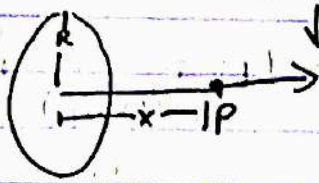
-when Gauss's Law fails, use Coulomb's Law!

$\vec{E}$ for Plane Charge (non-conducting)



σ - charge per unit area distributed through V
Calculate E

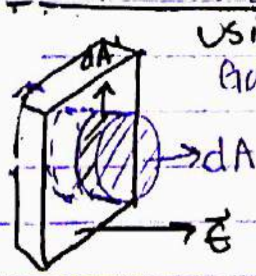
Recall... Coulomb's:



$$E_{disk} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{(x^2+R^2)^{1/2}}\right)$$

$R \to \infty$, like a $E_{infinite plane} = \frac{\sigma}{2\varepsilon_0}$

Using Gauss's Law:
Gaussian surface - cylinder



$$\Phi_{TOT} = \Phi_{ATot} + \cancel{\Phi_{A'Tot}}^{0}$$

$$= \oint_{ATot} \vec{E} \cdot d\vec{A} = \vec{E}\int_{ATot} d\vec{A} = 2\vec{E}\oint_{A} d\vec{A} = \underline{2EA}$$

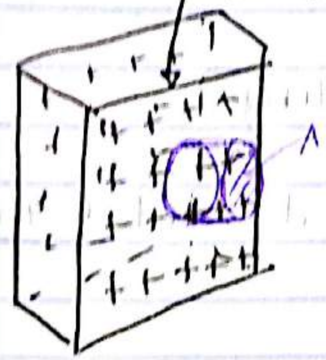
$$\frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} = 2EA$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

$\vec{E}$ for conducting plane

σ

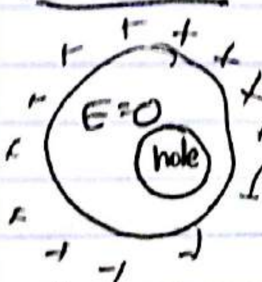
$\vec{E}=0$ inside



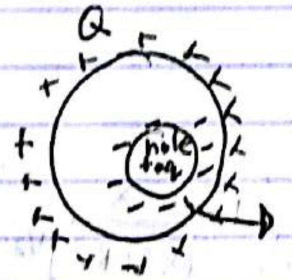
$$\Phi_E = EA = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}}$$

Conductor



charge will distribute itself so $\vec{E}_{inside}=0$.
(if not evenly dist, $F=qE$, so charges move.)



$\times P$ $E = ?$ $\propto \frac{(Q+q)}{\varepsilon_0}$ surface encloses Q & q.

$\vec{E}_{inside} = ?$
has to be 0.
conductors are lazy.
They don't like fields.