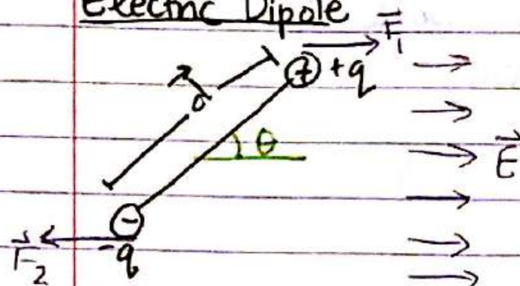
10/01/15

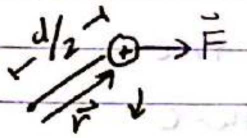
Electric Dipole



- dipoles are actually quite rigid.

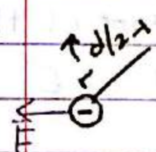
dipole moment, $\vec{p} = q\vec{d}$

These two forces $(\vec{F}_1, \vec{F}_2)$ exert a torque: $\tau = \vec{r} \times \vec{F}$



$$\tau_1 = \frac{d}{2} F \sin\theta$$
$$= \frac{dq}{2} E \sin\theta$$

$$\tau_T = \tau_1 + \tau_2$$
$$= q\vec{d}E\sin\theta$$
$$= \boxed{\vec{p} \times \vec{E}}$$



$$\tau_2 = \frac{d}{2} F \sin\theta$$
$$= \frac{dq}{2} E \sin\theta$$

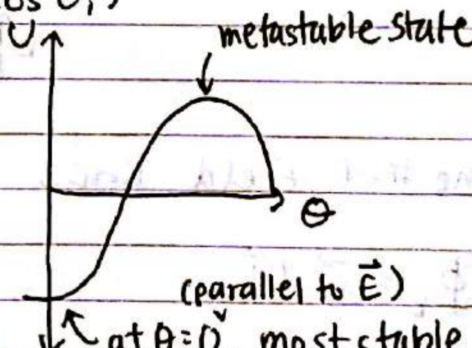
$$W = \int \vec{F}\, d\vec{r} = \int_{\theta_1}^{\theta_2} \tau\, d\theta = -\int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta$$
$$= pE(\cos\theta_2 - \cos\theta_1)$$

$\Delta U = -W = -pE(\cos\theta_2 - \cos\theta_1)$

$\theta_1 = 90^\circ,\ \theta_2 = \theta$

$U(\theta) = -pE\cos(\theta)$

$\rightarrow \boxed{U = -\vec{p} \cdot \vec{E}}$



(parallel to $\vec{E}$)

at $\theta = 0$, most stable

When the dipole starts oscillating, it emits light.

close to $\theta = 0$, like a parabolic func.

$\cos\theta \approx 1 - \frac{\theta^2}{2} + \dots$

$U \propto \theta^2$, so $F \propto -\theta$

this behaves like a spring (oscillation)

* this is also how microwaves work :)
  - $\vec{E}$ changes direction rapidly

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

surface integral ("closed integral")

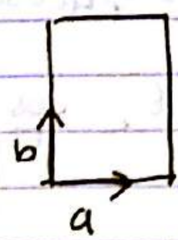
$\oint \vec{E} d\vec{A}$ is an integral over the surface of this sphere

It doesn't matter what the shape/size of the closed surface is, the same amt. of $\vec{E}$ passes through.

Flux: Φ

Think of a bucket.
$\Phi = \vec{A}\vec{v}$ - velocity of liquid entering Area

Area is a vector!



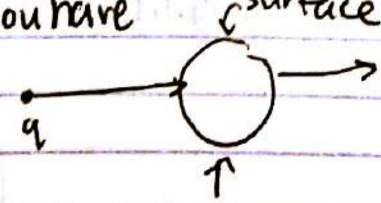
$\vec{A} = \vec{a} \times \vec{b}$
$|\vec{A}| = ab$

The # of field lines $\propto$ strength of $\vec{E}$ field

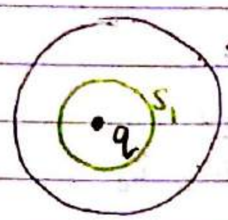
$\Phi_E = \vec{E} d\vec{A}$

ALL THE FLUX $= \oint \vec{E} \cdot d\vec{A}$

If you have



$\Phi_{NET} = 0.$
Need to enclose charge!



\# lines crossing S_1

= \# lines crossing S_2

\# lines crossing $S = \Phi_{S_1} = \underbrace{\text{line density}}_{\frac{\#\text{lines}}{\text{Area} \perp \text{to lines}}} \cdot \text{Area}$

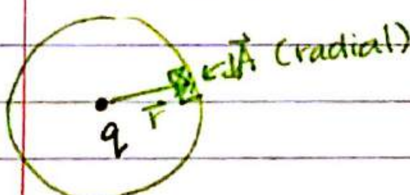
also $\propto \vec{E}$

$$\Phi_{S_1} = E \cdot \cancel{4\pi R^2} = \oint_{S_1} \vec{E} d\vec{A}$$

$$q / \cancel{4\pi\varepsilon_0 R^2} \qquad = \int_{S_2} \vec{E} d\vec{A}$$

thus, $\dfrac{q}{\varepsilon_0} = \int \vec{E} d\vec{A}$

<u>Derivation via Coulomb's law</u>



$$E = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$d\vec{A} = \hat{r} r^2 \sin\theta d\theta d\phi$ $E \cdot dA$

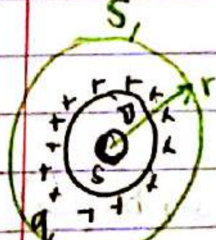
$[0,\pi]$ $[0, 2\pi]$

$$\oint d\vec{A} = r^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta\, d\phi$$

$$= 2\pi r^2 2 = 4\pi r^2$$

$$\vec{E} \cdot d\vec{A} = \vec{E} 4\pi r^2$$

$$\oint E \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

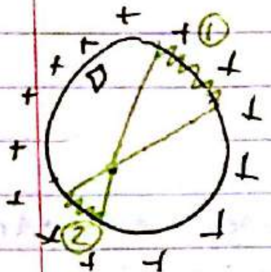


$\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc.} \leftarrow 0}{\varepsilon_0}$, $\boxed{E = 0 \text{ inside}}$

$$\oint_{S_1} \vec{E} \cdot d\vec{A} = q/\varepsilon_0$$

$$\vec{E} 4\pi r^2 = q/\varepsilon_0$$

$$\boxed{\vec{E} = q/4\pi\varepsilon_0 r^2 \,\hat{r}}$$

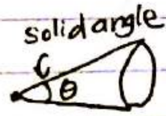


$$E_1 = \frac{q_1}{4\pi r_1^2 \varepsilon_0} \qquad E_2 = \frac{q_2}{4\pi r_2^2 \varepsilon_0}$$

$$= \frac{\sigma A_1}{4\pi r_1^2 \varepsilon_0} \qquad = \frac{\sigma A_2}{4\pi r_2^2 \varepsilon_0}$$

$$\frac{A_1}{r_1^2} \overset{?}{=} \frac{A_2}{r_2^2}$$

solid angle

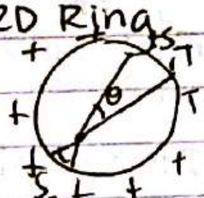


, solid angle same,

$$\frac{A_1}{r_1^2} = \frac{A_2}{r_2^2}.$$

Thought exp.

2D Ring



$q = 2\pi r \lambda$

$\theta = \theta$, so $\frac{S_1}{r_1} = \frac{S_2}{r_2}$

shitty conductor

outside surface - use coullomb's law.

inside surface - be careful - $\frac{r_1}{R}$

(assuming uniform distribution)

$$E = \frac{q}{4\pi\varepsilon_0 r_1 R}$$