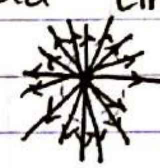
09/29/15

Review $\vec{E} = \frac{\vec{F}}{q}$ - only one charge

$\vec{F}$ - 2 charges

$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$

Field Lines



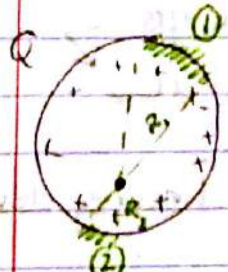
⁘ = field density
$(q>0)$

Q: what happens inside the conductor?

hollow, conducting sphere

A: There is no charge.
In a conductor, charge is outside the surface.
$\vec{E} = 0!$

$\vec{E} = 0$ at the center.



surface charge density: σ
$Q = \sigma \cdot$ Surface Area

▨ = contributing charge

$\vec{E}_1 = \frac{Q_1}{4\pi\varepsilon_0 R_1^2} = \frac{\sigma A_1}{4\pi\varepsilon_0 R_1^2}$

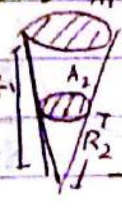
$\vec{E}_2 = \frac{Q_2}{4\pi\varepsilon_0 R_2^2} = \frac{\sigma A_2}{4\pi\varepsilon_0 R_2^2}$

$\vec{E}_{TOT} = \vec{E}_{1P} + \vec{E}_{2P}$

$\vec{E}_{TOT} = |\vec{E}_{1P}| - |\vec{E}_{2P}|$

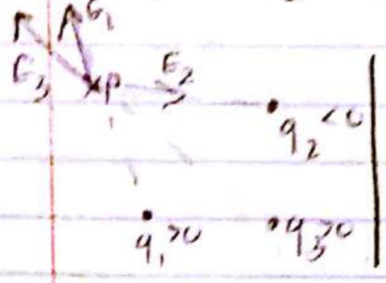
↳ are these equal?

$\frac{A_1}{R_1^2} \overset{?}{=} \frac{A_2}{R_2^2}$ ✓

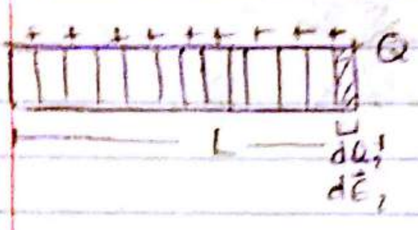


→ Geometric property of cones: $\frac{A_1}{R_1^2} = \frac{A_2}{R_2^2}$

Many charges → continuous distribution of charge



$$\vec{E}_{TOTP} = \sum_{i=1}^{3} \vec{E}_i$$



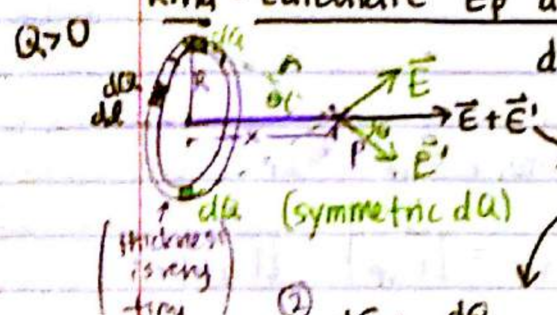
$$\vec{E}_{TOT} = \int_0^L d\vec{E}_i$$

RULES for a evenly distributed charged surface

1) Divide Q into infinitesimal dQ
2) Write Q in terms of charge density
   line - linear charge density λ, $Q = \lambda l$
   surface - surface charge density σ, $Q = \sigma A$
   Volume - volume charge density ρ, $Q = \rho V$
3) Write field associated with each dQ. (dE)
4) Total field = $\int dE$ can split up into components:
   ↳ $\int dE_x$, $\int dE_y$, $\int dE_z$

Ring - calculate $\vec{E}_P$ due to ring of Q, with linear chg. density λ

$dQ = \lambda dl$



① $\vec{E}_{TOTP} = \vec{E} + \vec{E}'$, $E_{TOTPy} = 0$, $E_{TOTPx} = \vec{E}_x + \vec{E}'_x$

- total field points along x-axis
  $Q>0$, $\odot \rightarrow \vec{E}$; $Q<0$, $\odot \leftarrow \vec{E}$

② $dE_P = \dfrac{dQ}{4\pi\varepsilon_0 r^2} = \dfrac{\lambda dl}{4\pi\varepsilon_0 r^2}$

③ $dE_{xP} = dE_P \cos\theta$

$\cos\theta = \dfrac{x}{r}$

$r = \sqrt{R^2+x^2}$

④ $E_{TOTP} = \int dE_{xP}$

⑤ $\displaystyle\int_0^{2\pi R} \frac{\lambda dl}{4\pi\varepsilon_0 (R^2+x^2)} \cdot \frac{x}{\sqrt{R^2+x^2}}$

⑥ $\displaystyle = \frac{\lambda x}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \int_0^{2\pi R} dl = \boxed{\frac{Qx}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}}}$

$\lambda 2\pi R$

Field of Ring @ Point P:

$$\frac{Qx}{4\pi\varepsilon_0(R^2+x^2)^{3/2}} \quad \text{if } x >> R,$$

$$= \frac{Qx}{4\pi\varepsilon_0 x^3} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

!! :)

↑ that $1/r^2$ dependence ♡ ♡

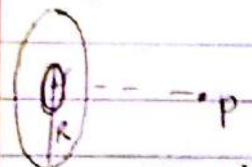
if $x << R$,

$$= \frac{Qx}{4\pi\varepsilon_0 R^3},$$

also if $x=0$, $E_{TOT}=0$.

↓ this is like a spring!, it will oscillate

DISK



$dQ = \sigma dA \quad A = \pi r^2 \quad dA = 2\pi r dr$

$$dE_{ring} = \frac{dQ\, x}{4\pi\varepsilon_0(x^2+r^2)^{3/2}}$$

no y-component

$$E_{TOTP} = \int_0^R dE_{ring}$$

$$= \int_0^R \frac{2\pi\sigma r dr x}{4\pi\varepsilon_0(x^2+r^2)^{3/2}}$$

↑ changes!

$$= \frac{2\pi\sigma x}{4\pi\varepsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

$$\int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

$$\int_0^R r(x^2+r^2)^{-3/2}\,dr$$

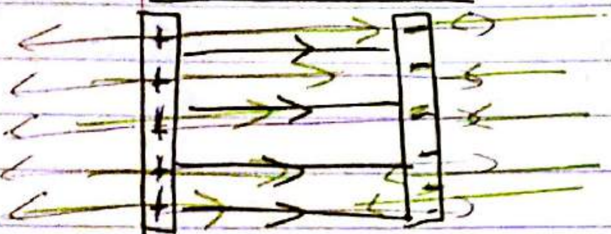
$$= -(x^2+r^2)^{-1/2}\Big|_0^R$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(x^2+R^2)^{3/2}}\right]$$

if $x=0$, $E = \frac{\sigma}{2\varepsilon_0}$

if $R \to \infty$, think of it as infinite sheet of charge

INFINITE PLATES



$$|E| = \frac{\sigma}{2\varepsilon_0}$$

$E_T = 0$ $\quad$ $E_T = E_+ + E_-$ $\quad$ $E_T = 0$

$$= \boxed{\frac{\sigma}{\varepsilon_0}}$$