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Chapter 4: Second Order DiffEqs

§4.1: Skipped

§4.2: Homogerous Linear Eqns

Differential equations: $\to$ ODE

① Ordinary diff. eq. - function of single variable

$y = y(t)$ (function of variable t)

Given an equation (equality relation) between

$y = y(t)$, $y' = y'(t)$, $y'' = y''(t)$ ...

The goal is to find $y = y(t)$.

e.g $y'(t) = y(t)$ (or $y' = y$)

Find y as a function of t.

Sol $y = e^t$ satisfies ✓

All solutions are of the form $\underbrace{y(t) = ce^t}_{\text{general solution}}$, c is constant

② Partial Diff Eq.

$y = y(t_1, t_2, \dots, t_n)$ $n \geq 1$.

functions of n variables $t_1, \dots, t_n$

Given eqn of partial derivatives,

The goal is to find all functions $y(t_1, \dots, t_n)$ satisfying the eqn.

e.g $y = y(t_1, t_2)$

$$\frac{\partial y}{\partial t_1} + \frac{\partial y}{\partial t_2} = 0$$

$\to y_{t_1} + y_{t_2} = 0.$

Sol
- constant func
- $x-y$ or $C(x-y)$
- e^{x-y}

$\to e^{x-y} - e^{x-y} = 0.$

This is too fancy

Actually, for any function $g(s)$ of a single variable, $y = g(x-y)$ satisfies eqn.

$\to$ Conclusion: PDE hard.

ODEs:

e.g.

$y' = 2y$ | $(e^{2t})' = 2e^{2t}$ ✓
So $y = Ce^{2t}$ is a gen. sol.

$y' = ay$ | $y = Ce^{at}$

$y' + by = 0$ | $y = Ce^{-bt}$.
$y' = -by$

$y'' - 3y' + 2y = 0$ | $y = e^t$ ✓ $\to$ Try $y = e^{rt}$
$\to r^2 e^{rt} - 3re^{rt} + 2e^{rt}$
$y = c_1 e^t + c_2 e^{2t}$ $= (r^2 - 3r + 2)e^{rt}$
$\to$ all sol. are of $= (r-2)(r-1)e^{rt}$
this form $\Rightarrow r = 1$ or 2

$y = c_1 e^t + c_2 e^{2t}$ is a linear combo of e^t, e^{2t}.

e.g. $y'' - 3y' + 2y = 0$

Homogeneous: "constant term" $= 0$.

Second-order: only have y, y', y''.

linear: LHS is a linear combo of y, y', y''.