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e.g. Write down the solution set of a system w/ augmented matrix

$$\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & 5 & 6 \\ & 1 & 3 & 0 & 4 & 7 \\ & & & 1 & 1 & 8 \end{array}\right] \leftarrow \text{already in reduced echelon form}$$

A general solution:

$$\begin{cases} x_1 = 6 - 2x_3 - 5x_5 \\ x_2 = 7 - 3x_3 - 4x_5 \\ x_3 = \text{free} \\ x_4 = 8 - x_5 \\ x_5 = \text{free} \end{cases}$$

can be written as a linear combo of vectors:

$$\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array}\begin{bmatrix} 6 \\ 7 \\ 0 \\ 8 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -5 \\ -4 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

constants + x_3 + x_5

$x_3, x_5 \in \mathbb{R}$

§1.7 Linear Independence

Dfn Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ be vectors in $\mathbb{R}^m$. These vectors are linearly dependent if there is a linear combination (with at least one nonzero constant) that equals 0.

Dfn If the system $x_1\vec{v}_1 + x_2\vec{v}_2 + \ldots + x_p\vec{v}_p = \vec{0}$ has only the trivial solution $x_1 = x_2 = \ldots = x_p = 0$, then the system is linearly independent.

Rmk. System is just

$$\begin{array}{ccc} A & \vec{x} & = \vec{0} \\ \underbrace{[\vec{v}_1, \ldots, \vec{v}_p]}_{m \times p \text{ matrix}} & \begin{bmatrix} x_1 \\ \vdots \\ x_p \end{bmatrix} & = \vec{0} \end{array}$$

e.g $\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\2\\-5\\8\end{bmatrix}, \begin{bmatrix}-1\\-1\\2\\11\end{bmatrix}$ linearly dependent?

Sol:

$$\begin{bmatrix}1 & 0 & -1\\2 & 2 & -1\\3 & -5 & 2\\4 & 8 & 11\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0 & -1\\0 & 2 & 1\\0 & -5 & 5\\0 & 8 & -15\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 0 & -1\\0 & 1 & 1/2\\0 & 0 & 15/2\\0 & 0 & -19\end{bmatrix}$$

Sol $\begin{bmatrix}0\\0\\0\end{bmatrix}$ ∴ linearly dependent.

e.g. $\vec{v}_1$ is linearly independent $\Leftrightarrow \vec{v}_1 \neq \vec{0}$

e.g. $\vec{v}_1, \vec{v}_2$ is linearly dependent if $x_1\vec{v}_1 + x_2\vec{v}_2 = 0$

$x_1 \neq 0$ $\left(\Rightarrow \vec{v}_1 = \frac{-x_2}{x_1}\vec{v}_1\right)$

OR

$x_2 \neq 0$ $\left(\Rightarrow \vec{v}_2 = \frac{-x_1}{x_2}\vec{v}_1\right)$

$\Leftrightarrow$ one vector is the scalar multiple of the other

§1.4 Linear Transformation

e.g. $\begin{cases} u = 2x+5y \\ v = -x+6y \end{cases}$

map from xy-plane to uv-plane.

vector function in calculus:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Let A be an $m \times n$ matrix. Then, the map

$$\mathbb{R}^n \longrightarrow \mathbb{R}^m$$

sending $\vec{x}$ to $A\vec{x}$ is a linear transformation

More general definition:

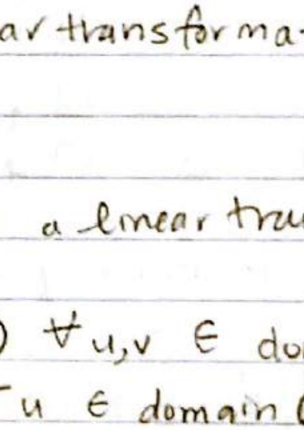
A map $T: \mathbb{R}^n \to \mathbb{R}^m$ is called a linear transformation if:

(i) $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ $\forall u,v \in$ domain(T)

(ii) $T(c\vec{u}) = cT(\vec{u})$ $\forall u \in$ domain(T), $c \in \mathbb{R}$.

Geometric pictures:

Solution set $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix}$



by multiplying by x_2, we get a line

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ offsets this line (translates it)