01/25

Choice of Free variables:

$x_1 + x_2 = 3$

$\left[\begin{array}{cc|c} 1 & 1 & 3 \end{array}\right]$, by convention, x_1 is pivot
x_2 is free variable.

Convert

$$\left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 9 & 16 & 25 \end{array}\right]$$

to reduced echelon form

Move (R1) to bottom (combination of interchange) ↓

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 9 & 16 & 25 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(R2) − (R1), (R3) − (R1) ↓

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 3 & 8 & 15 & 24 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

↑ C1 clear.

(R3) − 3(R2) ↓

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 2 & 6 & 12 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

↑ C2 Clear.

$\frac{1}{2}$(R3)

echelon form ↓

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(R3) − 3(R4), (R2) − 3(R4), (R1) − (R4)

$$= \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(R2) − 2(R3), (R1) − (R3) ↓

$$= \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

(R1) − (R2)

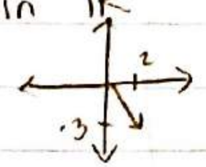
$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \checkmark$$

§1.3 Vector Equations

Vector in $\mathbb{R}^m$ is a column of m numbers
(an $m \times 1$ matrix)

e.g. vectors in $\mathbb{R}^2$

$$\begin{bmatrix} 2 \\ -3 \end{bmatrix}$$



Operations:

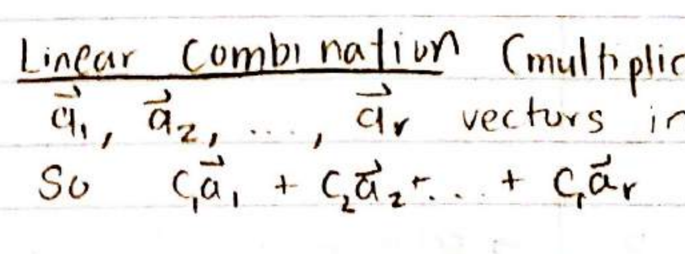
addition (component-wise)

$$\begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} a_1 + b_1 \\ \cdots \\ a_m + b_m \end{bmatrix}$$

scalar multiplication

$$c \cdot \begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} ca_1 \\ \vdots \\ ca_m \end{bmatrix}$$

Parallelogram rule



Linear combination (multiplication then addition)

$\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_r$ vectors in $\mathbb{R}^r$.

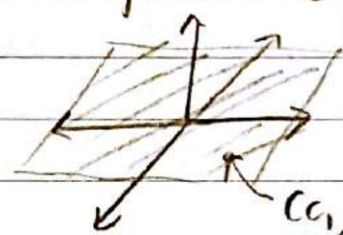
So $c_1\vec{a}_1 + c_2\vec{a}_2 + \ldots + c_r\vec{a}_r$ is a linear combination of $\vec{a}_1, \ldots, \vec{a}_r$.

The span of $\vec{a}_1, \ldots, \vec{a}_r$ is the set of all possible linear combinations of $\vec{a}_1, \ldots, \vec{a}_r$.

e.g. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$ in $\mathbb{R}^3$.

linear comb.: $c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}c_1\\c_2\\0\end{bmatrix}$.

span: c_1, c_2 are arbitrary.
And we get the plane $z=0$.



e.g. Is $\vec{b} = \begin{bmatrix}0\\5\\11\end{bmatrix}$ a vector in span $\{a_1, a_2\}$ for $\vec{a}_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$ $\vec{a}_2 = \begin{bmatrix}-3\\-1\\2\end{bmatrix}$?

linear comb: $\begin{bmatrix}c_1 + -3c_2\\2c_1 - c_2\\3c_1 + 2c_2\end{bmatrix}$

So... solve $\left[\begin{array}{cc|c}1 & -3 & 0\\2 & -1 & 5\\3 & 2 & 11\end{array}\right]$

$= \left[\begin{array}{cc|c}1 & -3 & 0\\0 & 5 & 5\\0 & 11 & 11\end{array}\right]$ → solution: $c_1 = 3$ ∴ Yes.
$c_2 = 1$.