

changes: HW 1: §1.3 is removed
Midterm II is now on 03/16 (not 03/18)

§1.2 continued

ex) Solve

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_2 - x_3 + x_4 = 2 \\ x_3 - x_4 = 0 \\ x_4 = 5 \end{cases}$$

(R4) gives x_4 . (R3) gives $x_3 = x_4 = 5$ (R2)... Trivial.

Idea: For a general system of equations, we use row operations to get to the point of the previous example.

Augmented matrices: (of ex. 1)

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{array} \right] \leftarrow \text{echelon form (upper triangular)}$$

it looks triangular

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -11 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 5 \end{array} \right] \leftarrow \text{reduced echelon form}$$

Def: A matrix is in echelon form if:

(i) All zero rows are below all nonzero rows

(ii) In every nonzero row, the leading nonzero entry (first nonzero element) is called a pivot entry.

* Require: Every entry below the pivot entry is zero.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -11 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 5 \end{array} \right] \quad \text{pivots}$$

(iii) Pivot entries go from upper left to lower right.

Reduced echelon form is in echelon form AND:

(i) All pivot entries are 1.

(ii) Entries above pivot are 0.

i.e. the pivot is the only nonzero entry of its column.

ex 2

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \left[\begin{array}{cccc|c} 5 & 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & 3 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{array} \right] & \text{* in} & \\ & & & & & \text{echelon form.} \end{array}$$

So ... $x_4 = -1$ but not enough information in (R1) to determine x_1 & x_5 .

$$x_3 = 4/3$$

$$\rightarrow 5x_1 + 5x_5 = -2$$

So there are infinitely many solutions. (it forms a line)

Solutions: $x_5 = \text{free}$ (free variable)

$$x_4 = -1$$

$$x_3 = 4/3$$

$$x_2 = \text{free}$$

$$x_1 = -2/5 - x_5$$

$$\Rightarrow \left\{ \begin{array}{l} \begin{bmatrix} -2/5 \\ 0 \\ 4/3 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \right\}$$

$x_5 = s, x_2 = t$

ex 3

$$\left[\begin{array}{ccccc|c} 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Solution

$$x_1 = \text{free}$$

$$x_2 = \text{free}$$

$$x_3 = 2$$

$$x_4 = \text{free}$$

$$x_5 = 5$$

Remark: (i) pivot variables are not free.

(ii) Any nonpivot variable is free.

General process to solve system of eqns:

- 1) Write augmented matrix
- 2) Change to a reduced echelon form by row operations
- 3) Recover the equations from the reduced echelon form
- 4) Write solutions (free variable = nonpivot)