01/21

Proving $0 \cdot v = 0 \Rightarrow \cancel{0 \cdot v} = (0+0)v = 0 \cdot v + \cancel{0 \cdot v}$

$\therefore 0 \cdot v = 0.$

Definition: V_1, V_2 vector spaces
$T: V_1 \rightarrow V_2$ is a linear transformation if:
i) $T(v_1 + v_2) = T(v_1) + T(v_2)$
ii) $T(\alpha v) = \alpha T(v)$

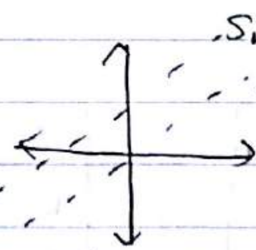
Ex $V_1 = \mathbb{R}^n$, $V_2 = \mathbb{R}^m$, A is an $m \times n$ matrix,
$T(x) := Ax$ is a linear transformation

abstract vector space is to $\mathbb{R}^n$ as abstract vector is to $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ as linear transformations are to matrices

Definition: V a vector space, $S \subseteq V$ is a subspace if it is also a vector space.
(subset, contained in)

$A \subseteq B$
$\forall a \in A, a \in B$

Ex. $V := \mathbb{R}^2$



$S_1 \rightarrow$ not subspace because does not contain 0
$S_2 \rightarrow$ a subspace!

Prop: $S \subseteq V$ is a subspace if:
i.) $0 \in S$ (S is not empty)
ii) $v_1, v_2 \in S \rightleftarrows v_1 + v_2 \in S$
iii) $v \in S, \rightleftarrows \alpha v \in S$

Definition: Let $T: V_1 \rightarrow V_2$ be a linear transformation
The kernal (nullspace) of T is:
$Ker(T) := \{x \in V_1 : T(x) = 0\}$ (set of solutions to $Ax = 0$)

Prop: $\ker(T)$ is subspace of V_1. $Im(T)$ is a subspace of V_2. (columnspace)

$A := \begin{bmatrix} 1 & 2 & 3 \\ -2 & -4 & -5 \end{bmatrix}$ $T: \mathbb{R}^3 \to \mathbb{R}^2$, $T(x) := Ax$

$\ker(T) = ? = \{x \in \mathbb{R}^3 : Ax = 0\}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -2 & -4 & -5 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \leftarrow \text{row echelon form}$$

, $x_3 = 0$.
$x_1 = 2x_2$

free variable pivots pivot column

$$= \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\ker(T) = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} : s \in \mathbb{R} \right\}$$

$(s = x_2)$

Ex:

$$\left[\begin{array}{ccccc|c} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

free variables

$$\left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 0 & -9 \\ 0 & 2 & -4 & 4 & 0 & -14 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$= \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \underbrace{\begin{bmatrix} -24 \\ -7 \\ 0 \\ 0 \\ 4 \end{bmatrix}}_{x_p} + s \underbrace{\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{x_{h_1}}, t \underbrace{\begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{x_{h_2}} \right.$$

the solutionset is a subspace translated by a vector

gen. sln = $x_p + x_h \rightarrow Ker(T_A)$ to $Ax=b$.