01/19
Disscussion

Def. A vector space is:

i) a set V with (vectors)

ii) a function $+ : V \times V \to V$, and

iii) a function $\cdot : \mathbb{R} \times V \to V$ such that

i) $+$ is associative — 4 - add

ii) $+$ is commutative — 2 - scalar mul. — 2 · both

iii) $\exists\, 0 \in V$ s.t. $0 + v = v = v + 0 \ \forall v \in V$ (there exists)

iv) $\forall v \in V$, $\exists!\, -v \in V$ s.t. $v + (-v) = 0 = (-v) + v$ (for all; there exists a unique vector)

v) $(\alpha \cdot \beta) \cdot v = \alpha \cdot (\beta \cdot v)$

vi) $1 \cdot v = v$

vii) $(\alpha + \beta) \cdot v = \alpha \cdot v + \beta \cdot v$

viii) $\alpha (v + w) = \alpha \cdot v + \alpha \cdot w$

$f : X \to Y$ — inc, dom, codom: can be larger than range

ex. $f : \mathbb{R} \to \mathbb{R}$, $f(x) := x^2$ (definition)

$\operatorname{Im}(f) = [0, \infty)$ $\operatorname{Cod}(f) = \mathbb{R}$ (real numbers)

$X \times Y := \{(x, y) : x \in X, y \in Y\}$ "cartesian product" (: such that; $\in$ for elem. of)

$\mathbb{R}^2 := \mathbb{R} \times \mathbb{R}$

Ex. $V = C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R}) := f : \mathbb{R} \to \mathbb{C} : f$ is smooth (inf. differentiable)

$[f + g](x) := f(x) + g(x)$

$[\alpha \cdot f](x) := \alpha f(x)$

Proposition: Let V be a vector space, $v \in V$, and $\mathcal{B} := \{b_1, \ldots, b_d\}$ be a basis of V. There $\exists!\, c_1, \ldots, c_d \in \mathbb{R}$ s.t.

$$v = c_1 b_1 + \ldots + c_2 d_2$$

$$[v]_{\mathcal{B}} := \begin{bmatrix} c_1 \\ \vdots \\ c_d \end{bmatrix}$$

$3x_1 - 2x_2 = 0$
$6x_1 + 4x_2 = 0$

Sol: $\left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : 3x_1 = 2x_2 \right\}$

$x_1 - 2x_2 - x_3 = 2$
$3x_1 - x_2 = 0$
$5x_1 - 2x_2 + x_3 = 1$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 3 & -1 & 0 & 0 \\ 5 & -2 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 5 & 3 & -6 \\ 0 & 8 & 6 & -9 \end{array}\right]$$ (mul row 1 by -3)

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 5 & 3 & -6 \\ 0 & 0 & 6-\frac{24}{5} & -9+\frac{18}{5} \end{array}\right]$$

$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} t : t \in \mathbb{R} \right\}$

$Ax = b$

$A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & -1 & 0 \\ 5 & -2 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $\{x \in \mathbb{R}^3 : Ax = b\}$

Thm: Let A be an $m \times n$ matrix, b an m-component column vector, and fix any particular solution $x_p \in \mathbb{R}^n$ (i.e. $Ax_p = b$), then $\forall$ solutions $x \in \mathbb{R}^n$, $\exists\, x_h \in \mathbb{R}^n$ w/ $Ax_h = 0$ s.t. $x = x_p + x_h$

$AX = B$ matrix eqn. $AX_1 = B_1$

$\Updownarrow$

$Ax = 0$