10/21/15

15.5 Applications of Double Integrals

Volume, Area - surface area covered in 15.6

Density & Mass

Suppose that a thin lamina has the shape of a region D in the xy-plane and has a density at (x,y) of $\rho(x,y)$. Then its total mass is $m = \iint_D \rho(x,y)\, dA$

Moments/Center of Mass

For the lamina in the region D and with density $\rho(x,y)$, the moment about the x-axis is:

$$M_x = \iint y\rho(x,y)\, dA$$ and the moment about y-axis:

$$M_y = \iint x\rho(x,y)\, dA$$

-because dist from (x,y) to x-axis is y
y-axis is x.

The coordinates of the center of mass $(\bar{x},\bar{y})$ of the lamina are: $\bar{x} = \frac{M_x}{m}$ and $\bar{y} = \frac{M_y}{m}$

- the lamina will balance horizontally if supported at $(\bar{x},\bar{y})$.

Moment of Inertia

The moment of inertia of our lamina about the x-axis is

$$I_x = \iint_D y^2\rho(x,y)\, dA$$ & moment ~~about~~ of inertia among y axis:

$$I_y = \iint_D x^2\rho(x,y)\, dA$$

about origin:

$$I_0 = \iint_D (x^2+y^2)\rho(x,y)\, dA = I_x + I_y$$

The radius of gyration of a lamina about an axis is the number R such that $mR^2 = I$

mass   moment of inertia

So, $m\bar{\bar{y}}^2 = I_x$ and $m\bar{\bar{x}}^2 = I_y$

Probability

A joint density function (of 2 variables) is a function $f(x,y)$ such that $f(x,y) \geq 0 \ \forall \ (x,y) \in \mathbb{R}^2$ and

$$\iint_{\mathbb{R}^2} f(x,y)\, dA = 1$$

improper integral (have to $R \to \infty$ lim)

If X, Y are continuous random variables, they have the joint density function $f(x,y)$ if the probability that (X,Y) lies in a region D is:

$$P((x,y) \in D) = \iint_D f(x,y)\, dA$$

If X is a random variable w/ density func $f_1(x)$ and y is a random variable w/ density func $f_2(y)$, then X & y are independent random variables and

$$f(x,y) = f_1(x) f_2(y)$$

Mean waiting time

$$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{\mu} e^{-t/\mu} & \text{if } t \geq 0 \end{cases}, \quad \mu \text{ is mean waiting time}, \quad \mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

If X, Y are rand. var. w/ joint density function $f(x,y)$, X-mean & Y-mean are:

$\mu_1 = \iint_{\mathbb{R}^2} x f(x,y)\, dA$ $\qquad \mu_2 = \iint_{\mathbb{R}^2} y f(x,y)\, dA$

"probability mass - similar to M_x & M_y"

A random variable is normally distributed, or it has normal distribution if its probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$ (μ = mean, σ = std. deviation)