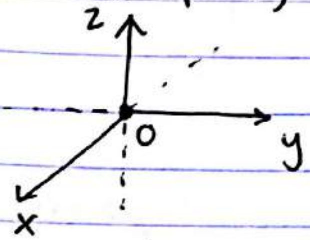
09/09/15

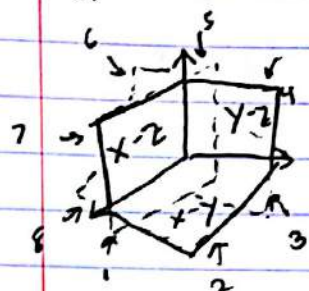
Lecture

12.1 3 Dimensional Coordinate Systems ← we're only talking about 1

- in a plane, we can identify a point with $\mathbb{R}^2$ (an ordered pair)
- in 3D space, we use $\mathbb{R}^3$ (ordered triple)



- use right-hand rule for axis convention



x-y : z = 0
x-z : y = 0
y-z : x = 0

- space is divided into 8 octants.

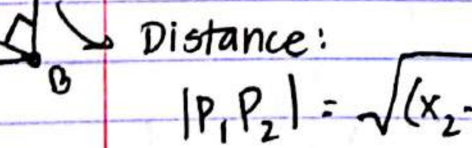
NOTE

you cannot tell the coordinates of P without further information

* exercise 39

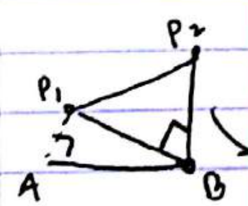
both are possible.

Given a point P,
a = directed distance from y-z plane to P



$\perp$ to x-y, x-z, and y-z plane
↓ c, ↓ b, ↑ a

$P = (a, b, c)$

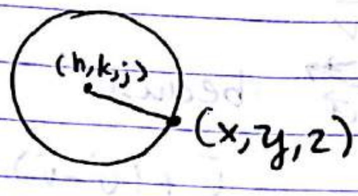


Distance:

$$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

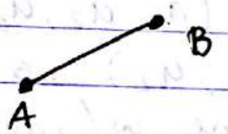
∴ eqn of a sphere:

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

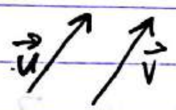


12.2 Vectors

- magnitude & direction

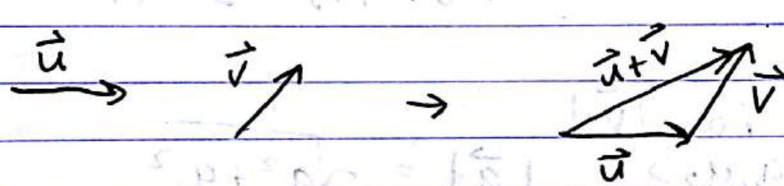


$\vec{V} = \vec{AB}$



if $\vec{U}$ and $\vec{V}$ have the same direction and magnitude, $\vec{u} = \vec{v}$.

vector Addition:



- sum x & y components
- commutative operation

Scalar multiplication
no direction

If λ is a scalar and $\vec{V}$ is a vector, then $\lambda\vec{V}$

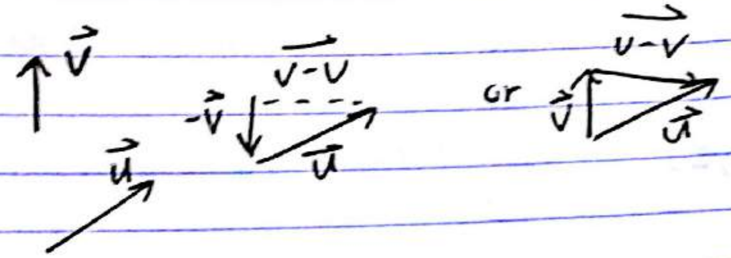
- length is $|\lambda|$ times the length of $\vec{V}$
- same direction as $\vec{v}$ if $\lambda > 0$, opposite if $\lambda < 0$
- if $\lambda = 0$ or $\vec{v} = \vec{0}$, then $= \vec{0}$

$-\vec{V}$ = same length, opposite directions

Two nonzero vectors are parallel if they are scalar multiples.

subtraction

$\vec{U} - \vec{V} = \vec{U} + (-\vec{V})$



because

$\vec{v} + (\vec{u} - \vec{v}) = u$

components of $\vec{a}$

place initial at 0

terminal point has (a_1, a_2) or (a_1, a_2, a_3)

NOTE: $\langle a_1, a_2 \rangle$ is vector, (a_1, a_2) is a point

infinite representations of $\vec{a}$, but one w/ initial point @ 0 is position vector.

Given points $A(x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$

vector $\vec{u}$ $(\vec{AB})$ is:

$$\vec{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Magnitude

length of $\vec{v}$ is $|\vec{v}|$

If $\vec{a} = \langle a_1, a_2 \rangle$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$

Algebraic Arithmetic

$\vec{a} = \langle a_1, a_2 \rangle \quad \vec{b} = \langle b_1, b_2 \rangle$

$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$\lambda \vec{a} = \langle \lambda a_1, \lambda a_2 \rangle$$

(same applies for a 3D vector).

$V_n = \langle a_1, a_2, \ldots a_n \rangle$

Properties

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

3. $\vec{a} + 0 = \vec{a}$

4. $\vec{a} + (-\vec{a}) = 0$

5. $\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$

6. $(\lambda + \mu)\vec{a} = \lambda\vec{a} + \mu\vec{a}$

7. $(\lambda\mu)\vec{a} = \lambda(\mu\vec{a})$

8. $1\vec{a} = \vec{a}$

Standard vectors

In $\mathbb{R}^2$, $\hat{i} = \langle 1, 0 \rangle$, $\hat{j} \cdot \langle 0, 1 \rangle$

In $\mathbb{R}^3$, $\hat{i} = \langle 1, 0, 0 \rangle$, $\hat{j} = \langle 0, 1, 0 \rangle$, $\hat{k}$ $\langle 0, 0, 1 \rangle$

In $\mathbb{R}^2$, $\vec{a} = \langle a_1, a_2 \rangle = a_1\hat{i} + a_2\hat{j}$

In $\mathbb{R}^3$, $\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Unit vector

$\vec{V}$ with $|\vec{V}| = 1$

- $\hat{i}, \hat{j}, \hat{k}$ are unit vectors

if $\vec{a} \neq \vec{0}$, then unit vector w/same direction as $\vec{a}$ is:

$\vec{u} = \frac{1}{|\vec{a}|} \cdot \vec{a} = \frac{\vec{a}}{|\vec{a}|}$

☆ be careful w/units.

$\vec{F} = m\vec{g} = \frac{kg \cdot m}{s^2}$