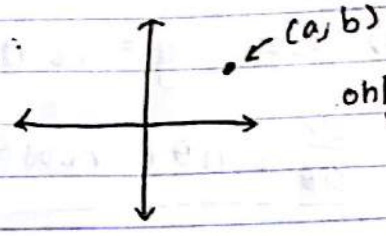
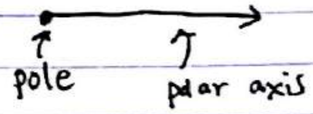
08/31/15

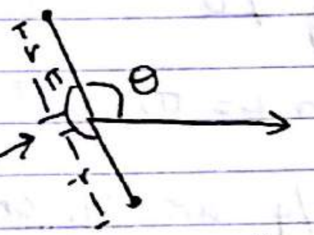
10.4 Lecture

Cartesian:



only one ordered pair per point





(r, θ)

- If $P=0$, $(0, \theta)$ will represent pole of every value of θ.
- The angle is measured counterclockwise
- r can be negative if:
  $(-r, \theta) = (r, \theta+\pi)$

Infinite representations of a single point:

$(r, \theta) = (r, \theta+2n\pi) = (-r, \theta+(2n+1)\pi)$

Cartesian → Polar

$x = r\cos\theta$ $\quad\quad r^2 = x^2+y^2 \rightarrow r = \sqrt{x+y}$

$y = r\sin\theta$ $\quad\quad \tan\theta = y/x \rightarrow \theta = \arctan(y/x)$

make sure θ is in correct quadrant

↳ $\tan\theta$ has period π!

Polar Curves

$r = f(\theta)$ or $F(r, \theta) = 0$

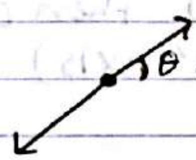
$\theta = k$

$\theta = k$

$\tan\theta = \tan k$

$\frac{y}{x} = \tan k$

$y = x\tan(k)$

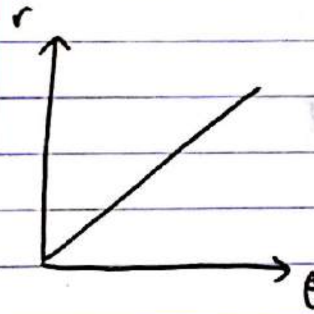


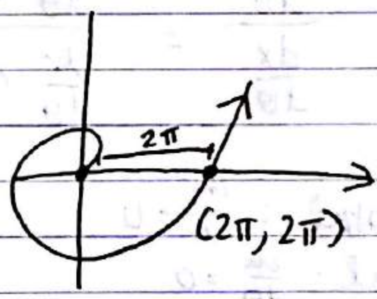
Helpful Techniques

Plot r vs. θ on cartesian coordinates

example 33

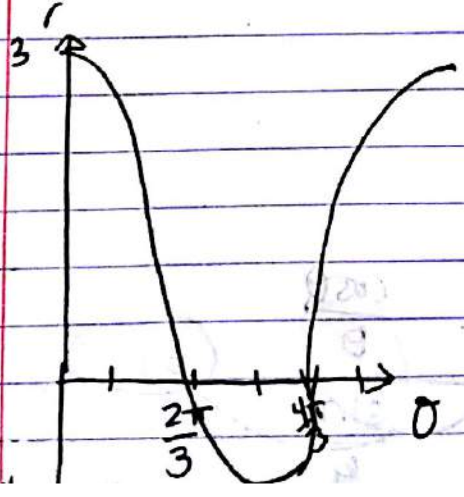
$r = \theta, \ \theta \geq 0$

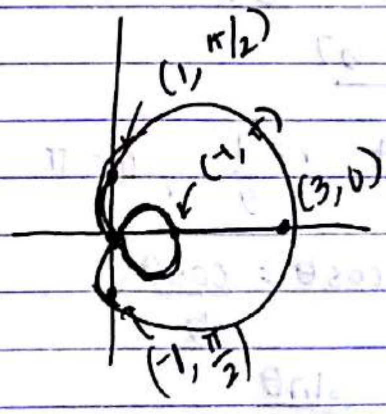




example 32

$r = 1 + 2\cos\theta$





Symmetry

If $F(r,\theta) = F(r,-\theta)$, then curve is symmetric about polar axis ($\longrightarrow$)

If $F(r,\theta) = F(r,\pi-\theta)$ then curve is symmetric about $\theta = \frac{\pi}{2}$ (vertical axis) $\uparrow$

If

Tangents

If $r = f(\theta)$, regard θ as a parameter

$x = r\cos\theta = f(\theta)\cos(\theta)$

$y = r\sin\theta = f(\theta)\sin\theta$

therefore:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

horizontal : $\frac{dy}{d\theta} = 0$

vertical : $\frac{dx}{d\theta} = 0$

If $\frac{dy}{dx} = \frac{0}{0}$, L'Hospital

example 67

tangent $r = \frac{1}{\theta}$, $\theta = \pi$

$x = r\cos\theta = \frac{\cos\theta}{\theta}$

$y = \frac{\sin\theta}{\theta}$

$$- \frac{\frac{\cos\theta}{\theta}}{\frac{\sin\theta}{\theta}} =$$

example 51 cont.

$$\frac{dy}{dx} = \frac{\frac{-1}{\theta^2}\cdot\sin\theta + \frac{1}{\theta}\cos\theta}{\frac{-1}{\theta^2}\cos\theta + \frac{-1}{\theta}\sin\theta}\left(\frac{\theta^2}{\theta^2}\right)$$

$$= \frac{-\cancel{\sin\theta} + \theta\cos\theta}{-\cos\theta - \cancel{\theta\sin\theta}} \qquad \theta = \pi$$

$$= \frac{-\pi}{1} = -\pi$$

yay!

ex: 52

Sketch $(x^2+y^2)^3 = 4x^2y^2 \qquad r^2 = x^2+y^2$

$$r^6 = 4r^2\sin^2\theta\, r^2\cos^2\theta$$

$$r^2 = 4\sin^2\theta\cos^2\theta$$

$$\boxed{r = \pm 2\sin\theta\cos\theta} = \sin(2\theta)$$

θ	
0	0
$\frac{\pi}{2}$	0
π	0
$\frac{3\pi}{2}$	0
2π	0
$\frac{\pi}{4}$	1
$\frac{3\pi}{4}$	-1