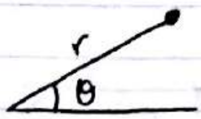
08/30/15

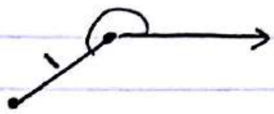
10.3 Polar Coordinates



(r, θ)

example 1

ⓐ $(1, 5\pi/4)$



- polar coords have multiple expressions.

$(1, 5\pi/4) = (1, -3\pi/4) = (1, 13\pi/4)$ and so on

$(r, \theta) = (r, \theta + 2n\pi) = (-r, \theta \pm (2n+1)\pi)$

Cartesian to Polar:

$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$

$\therefore \quad x = r\cos\theta \qquad y = r\sin\theta$

$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$

example 2

$(2, \pi/3)$ to cartesian

$x = \cancel{\cos\theta}\ r\cos\theta \qquad y = r\sin\theta$

$= 1 \qquad = \sqrt{3}$

$= \boxed{(1, \sqrt{3})}$

example 3

$(1,-1)$ to polar

$\because$ Q IV $\therefore$ $\theta = -\pi/4$

$r = \sqrt{2}$ $\tan\theta = -1$

$$\boxed{(\sqrt{2},\ 7\pi/4)}$$

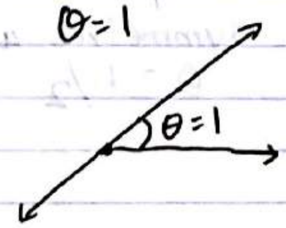
Polar Curves

$r = f(\theta)$ or $F(r,\theta) = 0$??

the eqn $r=2$ gives circle w/radius 2.

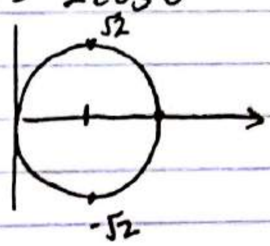
example 5

$\theta = 1$



example 6

$r = 2\cos\theta$



$x = r\cos\theta$

$\frac{x}{r} = \cos\theta$

$r = \frac{2x}{r}$

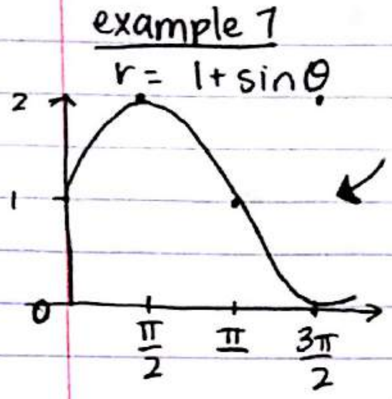
$r^2 = 2x = x^2 + y^2$

$\therefore \boxed{-2x + x^2 + y^2 = 0}$

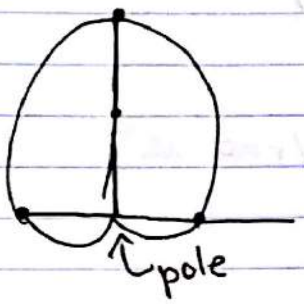
$= (x-1)^2 + y^2 = 1$

example 7

$r = 1 + \sin\theta$

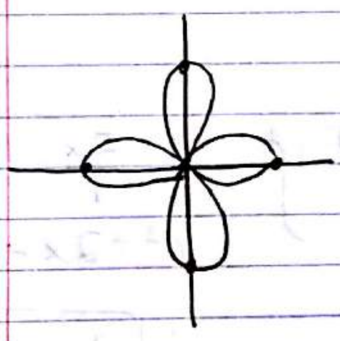


cartesian graph



example 8

$r = 3\cos 2\theta$



Symmetry

(a) If a polar equation is not changed when θ is replaced with $-\theta$ it is symmetric to polar (θ) axis.

(b) If the equation is unchanged when r is replaced with $-r$ or when θ is replaced by $\theta + \pi$ is symmetric about pole

(c) if unchanged wh θ replaced by $\pi - \theta$, symmetric about $\theta = \pi/2$

Tangents to Polar Curves

$x = r\cos\theta = f(\theta)\cos\theta \qquad y = r\sin\theta = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

tangents @ $r=0$:

$$\frac{dy}{dx} = \tan\theta \quad \text{if } \frac{dr}{d\theta} \neq 0$$

ex. $r = \cos 2\theta = 0$ when $\theta = \pi/4$ or $3\pi/4$

$\theta = 3\pi/4$ & $\theta = \pi/4$ are tangent

example 9

$r = 1 + \sin\theta$, tangent when $\theta = \pi/3$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{\cos\theta\sin\theta + (1+\sin\theta)\cos\theta}{\cos\theta\cos\theta - (1+\sin\theta)\sin\theta}$$

$$= \frac{\cos\theta(1+2\sin\theta)}{(1+\sin\theta)(1-2\sin\theta)}$$