10.2 Lecture (08/28)

Tangents

$x = f(t) \qquad y = g(t) \qquad y = F(x)$

$g'(t) = F'(f(t)) \cdot f'(t)$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$ wow!

tangent horizontal when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$

tangent vertical when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

Concavity

$F''(x)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

Area

$$A = \int_a^b y\,dx = \int_\alpha^\beta g(t)\, f'(t)\, dt$$

☆ make sure α & β are in terms of t

Arc Length

do $\frac{\Delta x}{\Delta y}$ lots of times $\left(\sqrt{x^2+y^2}\right)$

thus,

$$L = \int_a^b ds$$

$$\frac{\Delta s_i}{\Delta x} = \frac{\sqrt{\Delta x^2 + \Delta y_i^2}}{\Delta x} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

need in terms of x, not s

Parametric:

$$L = \int_\alpha^\beta \sqrt{1 + \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)^2} \frac{dx}{dt} \, dt$$

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

valid when f' and g' are continuous

Surface Area - review book!

flatten

$$A = \frac{1}{2} l^2 \theta = \frac{1}{2} l^2 \left(\frac{2\pi r}{l}\right)$$

↓

sector

$$A = \pi r l$$

$$A = 2\pi r l, \quad r = \frac{1}{2}(r_1 + r_2)$$

subtract smaller cone

Rotate C about x-axis

$$S \approx \sum A_i$$

$$A_i = 2\pi\left(\frac{y_{i-1} + y_i}{2}\right)\left|P_{i-1} - P_i\right|$$

$$\left|P_{i-1} - P_i\right| = \sqrt{1 + F'(x_i)^2}\, dx$$

$$\therefore S = \int_a^b 2\pi F(x)\sqrt{1 + F'(x)^2}\, dx$$

$$= \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_a^b 2\pi y\, ds$$

Parametric:

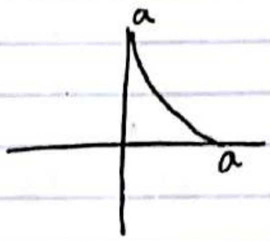
$x = f(t) \quad y = g(t)$

$$S = \int_{\alpha}^{\beta} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

<u>exercise 34</u>

$x = a\cos^3\theta \quad y = a\sin^3\theta$

$$A = 4\int_0^a y\,dx$$



$$A = 4\int_0^{\alpha} y(\theta)\cdot x'(t)\, dt$$

$$= -4\int_0^{\alpha} a\sin^3\theta \cdot 3a\cos^2\theta \sin\theta\, d\theta$$

$$= -4a\int_0^{\alpha} \sin^3\theta \cos^2\theta \sin\theta\, d\theta \quad , \quad \alpha = 0, \ \theta = \frac{\pi}{2}$$

$$= 12a^2\int_0^{\pi/2} \sin^3\theta \cos^2\theta \sin\theta\, d\theta$$

$$= 12a^2\int_0^{\pi/2} \underline{\sin^4\theta \cos^2\theta}\, d\theta$$

$\sin^2\theta \sin^2\theta \cos^2\theta$

$= \frac{1}{4}\sin^2\theta \cdot \sin^2\theta\, d\theta$

$$A = 3a^2 \int_0^{\pi/2} \sin^2\theta \sin^2\theta \, d\theta$$

$$= \frac{3a^2}{2} \int_0^{\pi/2} (1-\cos 2\theta) \sin^2 2\theta \, d\theta$$

$$=$$

Surface Area (from Book)

example 6

$x = r\cos t \quad y = r\sin t$

$$S = \int_0^{\pi} 2\pi r \sin t \sqrt{(-r\sin t)^2 + (r\cos t)^2}\, dt$$

$$S = 2\pi \int_0^{\pi} r^2 \sin t \, dt$$

$$S = 2\pi r^2 \int_0^{\pi} \sin t \, dt$$

$$= 2\pi r^2 \left[\cancel{\sin} - \cos t \right]_0^{\pi} \; 0$$

$$= \boxed{4\pi r^2}$$