

Office Hrs Matt Tanzer: MW 1-2pm 739 Evans

### 10.1 Curves Defined By Parametric Equations

- parametric equations operate by a function of time.
- make parametric curve.

#### EXAMPLE 1

$$x = t^2 - 2t \quad y = t + 1$$

$\uparrow$   
\_\_\_\_\_  $t = y - 1$

$$x = (y - 1)^2 - 2(y - 1)$$

$$= y^2 - 2y + 1 - 2y + 2$$

$$x = y^2 - 4y + 3$$

\* there are also parametrics w/ limits.

#### EXAMPLE 2/3

$$x = \cos(t) \quad y = \sin(t) \quad 0 \leq t \leq 2\pi$$

$$x^2 + y^2 = \sin^2(t) + \cos^2(t) = 1$$

→ same as

$$x = \cos(2t) \quad y = \sin(2t) \quad 0 \leq t \leq 2\pi$$

#### EXAMPLE 4

A circle with center  $(h, k)$  & radius  $r$

$$x = r\cos(t) + h \quad y = r\sin(t) + k \quad 0 \leq t \leq 2\pi$$

EXAMPLE 5

$$x = \sin(t)$$

$$y = \sin^2 t$$

$$y = x^2 \leftarrow \text{parabola!!}$$

EXAMPLE 6

$$x = y^4 - 3y^2$$

$$y = t$$

$$x = t^4 - 3t^2$$

$$y = t$$

cheat:

if  $x = g(y)$

just graph  $y = t$ ,  $x = g(t)$

if  $y = f(x)$

just graph  $x = t$ ,  $y = f(t)$

EXAMPLE 8

$$x = a + \cos(t)$$

$$y = a \tan(t) + \sin(t)$$

$\rightarrow$  conchoids of Nicomedes

10.1 Lecture (08/29/15)

- Div, Grad, Curl (rec. book)
- Dec 14 - Final 8am - 11am

It is important to note how a curve is traced out.

ex.  $x = \sin(t)$   $y = \cos(t)$

vs.

$x = \cos(t)$   $y = \sin(t)$

↳ these start @ diff points, but make same curve.

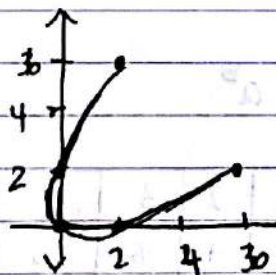
with a parameter restriction ( $t \in [a, b]$ ), you get an initial & terminal point.

\*  $\forall t =$  "for all  $t$ "

exercise!

$x = t^2 + t$ ,  $y = t^2 - t$

$t \in [-2, 2]$



\* it dips below!

t	x	y
-2	2	6
-1	0	2
0	0	0
1	2	0
2	6	2

$x - y = 2t$ ,  $x + y = 2t^2$

$B^2 - 4AC = (-2)^2 - 4 \cdot 1 \cdot 1 = 0$

$(x - y)^2 - 2(x + y) = 0$

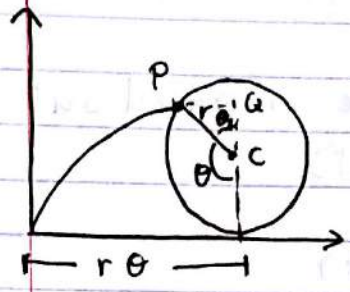
$\cot(2\theta) = \frac{C - A}{B} = 0 = \frac{\pi}{4}$

$x^2 - 2xy + y^2 - 2x - 2y = 0$

(45°)  
rotation

parabola

exercise 39



$$\theta_2 = \pi - \theta$$

$$C = (r\theta, r)$$

$$P = (r\theta, r\cos\theta)$$

Yah! Geometry!

$$Q = (r, r - r\cos\theta)$$

$$PQ = r\theta + r\sin(\pi - \theta)$$

$$= r(\theta - \sin\theta) \leftarrow x$$

what's bigger?

$e^\pi$  or  $\pi^e$  ?  
 $(2.7)^{3.14}$      $(3.14)^{2.7}$

$\hookrightarrow a^b$

	b			
a	1	2	3	4
2	2	4	8	16
3	3	9	27	81
4	4	16	64	256

$1^{10} = 1$      $2^{10} = 1024$      $3^{10} = 59049$      $4^{10} = 1048576$   
 $10^1 = 10$      $10^2 = 100$      $10^3 = 1000$      $10^4 = 10,000$

$1^9 = 1$      $2^5 = 32$      $3^5 = 243$      $4^5 = 1024$   
 $5^1 = 5$      $5^2 = 25$      $5^3 = 125$      $5^4 = 625$

as result of this trend,  $e^\pi$  is larger.

$$\begin{array}{r} 13 \\ \times 128 \\ \hline 512 \end{array}$$

$$\begin{array}{r} 256 \\ \times 4 \\ \hline 512 \end{array}$$