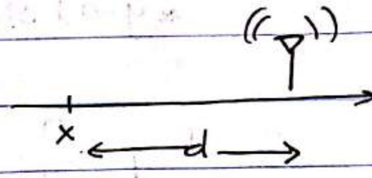
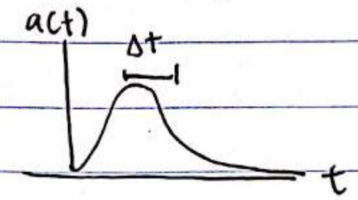
Positioning



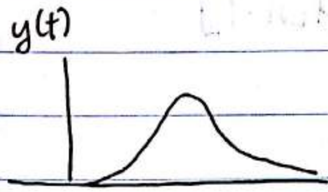
Time of Flight (ToF)



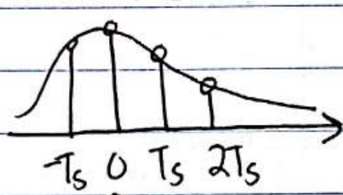
$d = v\Delta t$

$v = 3e8\,m/s$ (radio/EM)

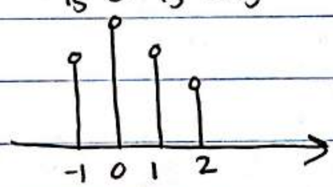
$v = 330\,m/s$ (audio)



Discrete Time:



$a_{CT}(t)$



$a_{DT}[n] = a_{CT}(nT_s)$

T_s: (sec/sample)

f_s : $1/T_s$ (cycles/sec)

$y[n] = \alpha a[n - N_a]$ · Determine the delay N_A samples

$\Delta t = N_a T_s$.

$d = vN_aT_s$.

How do we find N_A?

– Beacon signal is periodic w/ period N (N-periodic)

$a[n-N] = a[n] \quad \forall n \in \mathbb{Z}$

$y[n] = \alpha\, a[n - N_A]$ $\quad$ * $y[n]$ still has same period.

$$\vec{y} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} \qquad \vec{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \leftarrow a[n]$$

$$\downarrow$$

$$\begin{bmatrix} a_2 \\ a_0 \\ a_1 \end{bmatrix} \leftarrow a[n-1]$$

scalar 1

$$\begin{bmatrix} a_2 \\ a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \underbrace{}_{}\overset{\vec{a}}{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}$$

I_2

$$\begin{bmatrix} a_{N-1} \\ a_0 \\ \vdots \\ a_{N-2} \end{bmatrix} = \underbrace{\begin{bmatrix} 0^T_{N-1} & 1 \\ I_{N-1} & \vec{0}_{N-1} \end{bmatrix}}_{S} \begin{bmatrix} a_0 \\ \vdots \\ a_{N-1} \end{bmatrix}$$

$\vec{y} = S^{N_A}\vec{a}$ $\qquad$ $S^N = I_N$ (period)

We don't know N_A. To determine it, look @ all possible shifts $S^k \vec{a}$ ($k \in [0, N)$) and find value of k that gives the "closest match".

$\longrightarrow$ projection! (innerproduct)

maximize $|\langle \vec{y}, S^k \vec{a} \rangle|$.

Inner Product : $x, y, z \in V$ ($\mathbb{R}^n$ or $\mathbb{C}^n$)

$\langle \; , \; \rangle : V \times V \longrightarrow \mathbb{R}$ or $\mathbb{C}$

$\langle x, y \rangle = \langle y, x \rangle^*$ Hermitian Symmetry

$\langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle$ distributivity

$\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$ scaling

$\langle x, x \rangle \geq 0$ w/equality iff $x = 0$ (nonnegativity)

$\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}^* \longrightarrow$ In Math 54,

$\langle \vec{x}, \vec{y} \rangle = (\vec{x}^T)^* \vec{y}$

$= \vec{x}^H \vec{y}$

for $\mathbb{R}^n$,

$\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y} = \vec{x} \cdot \vec{y}$

Norm: a sense of size

$\|\vec{x}\| \geq 0$ w/ equality iff $\vec{x}$ is $\vec{0}$.

$\|\alpha \vec{x}\| = \alpha \|\vec{x}\|$ scaling

$\|\vec{x} + \vec{y}\| \leq \|x\| + \|y\|$ w/equality iff $\vec{x} = a\vec{y}$ (parallel)

triangle equality

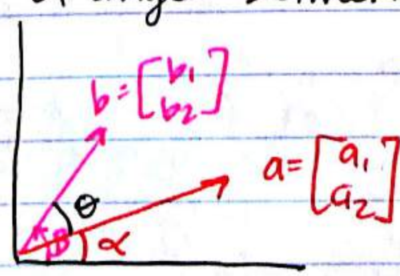
The norm we use is based on inner product:

$\|\vec{x}\| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$, $\|\vec{x}\|^2 = \langle \vec{x}, \vec{x} \rangle$

2-Norm

$\|\vec{x}\|_2$

cos of angle between vectors



$a_1 = \|\vec{a}\| \cos\alpha$ $\quad b_1 = \|\vec{b}\| \cos\beta$

$a_2 = \|\vec{a}\| \sin\alpha$ $\quad b_2 = \|\vec{b}\| \sin\beta$

$\theta = \beta - \alpha$ $\quad \langle a, b \rangle = \cos\theta \|a\| \|b\|$

(use trig id)

Example

$$\vec{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \qquad \begin{matrix} \vec{a}[n-1] \\ S\vec{a} \end{matrix} \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} \qquad \begin{matrix} \vec{a}[n-2] \\ S^2\vec{a} \end{matrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

$\langle \vec{a}, \vec{a} \rangle = 7$

$\langle \vec{a}, \vec{a}[n-1] \rangle = -1$

$\langle \vec{a}, S^2\vec{a} \rangle = -1$