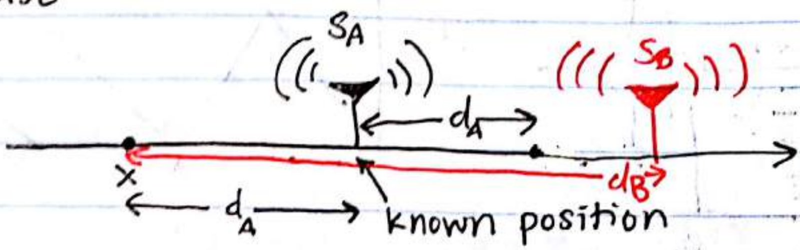
03/29

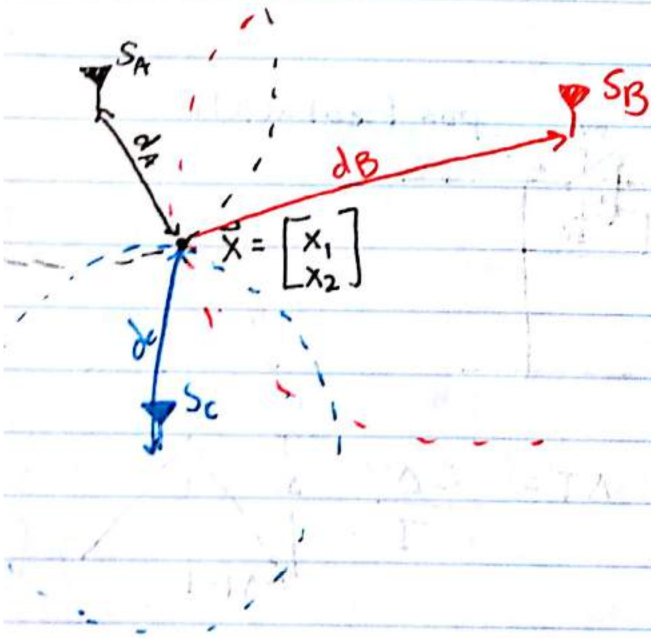
Positioning: "Acoustic locationing"

1-D CASE




ambiguity! How do we resolve? (another station B)

2-D CASE




$\vec{x}$ is at intersection of all three circles

*the 1-D case is a special case of 2-D case

How do we determine the distances?

- assume signal from the beacon travels @ $c = 3e8$ m/s
  - need to know when signal is sent (ref. time) $t=0$
  - clocks must be synchronized

Let $\Delta t = 1\mu sec$ clock offset between beacon + receiver?

→ dist. estimate will be 300 m off ($c\Delta t = 300m$)

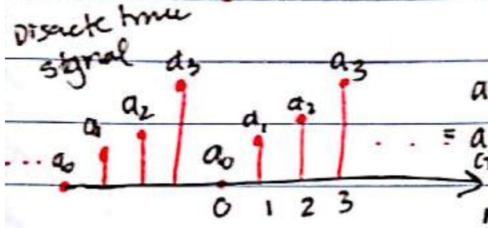
This is called time-of-arrival (ToA) positioning.

In lab, you will do time-difference-of-arrival (TDoA) positioning.


$a(t)$ $\quad a_{CT}: \mathbb{R} \to \mathbb{R}$ continuous-time-signal

analog signal

$T_s$: sampling period (sec)




$a[n] = a_{CT}(nT_s)$ $\quad a : \mathbb{Z} \to \mathbb{R}$ (integers)

$T_s$ : sec $\rightarrow f_s = \frac{1}{T} Hz$

$f_s = 44.1 kHz$

$a[0] = a_0 \leftrightarrow a[1] = a_1$

$T_s = \frac{1}{f_s} = 0.0000226$ sec apart

($22.6 \mu sec$)

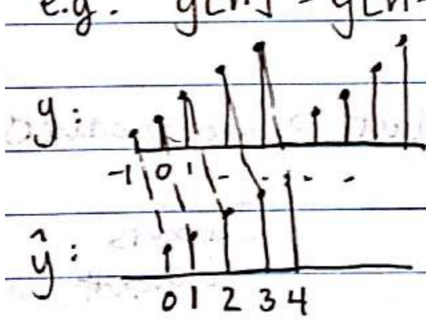
what if the received signal is $a[n-5000] = \hat{a}[n]$

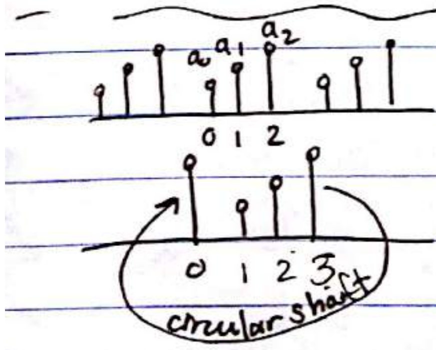
$\hat{a}[n]$ is $a[n]$ right-shifted by 5000 $T_s$.

or a 5000 sample delay.

e.g. $\hat{y}[n] = y[n-1]$

$\hat{y}[n] = y[n]$ delayed by 1 $T_s$.




$a[n]$

$a[n-k]$, $k=1$

$$a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{N-1} \end{bmatrix} \xrightarrow{1 \text{ delay}} \begin{bmatrix} a_{N-1} \\ a_0 \\ \vdots \\ a_{N-2} \end{bmatrix}$$

$$\begin{bmatrix} a_4 \\ a_0 \\ a_1 \\ a_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{S} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$\underline{\vec{a}}$   $\underline{S\vec{a}}$   $S^2\vec{a}$

$a[n]$   $s[n-1]$   $a[n-2]$

$$\vec{y} = \alpha \underbrace{S^{N_a}\vec{a}}_{a[n-N_a]} + \beta \underbrace{S^{N_b}\vec{b}}_{b[n-N_B]} + \gamma \underbrace{S^{N_c}\vec{c}}_{c[n-N_c]}$$

received signal

We must find $N_a$, $N_b$, and $N_c \longrightarrow$ delay $_A$, delay$_B$, delay$_C$

$f_s$ ↓ speed of wave

$d_A$, $d_B$, $d_C$

We want $S^{N_a}\vec{a}$, $S^{N_b}\vec{b}$, $S^{N_c}\vec{c}$ to be orthogonal.

and all shifts of a to be orthogonal

-but that's too much orthogonal

To measure "similarity", we use dot products → inner products

$\vec{x} \cdot \vec{y} = \vec{x}^T\vec{y}$

$\langle \vec{x}, \vec{y} \rangle = xy^*$ ← complex conjugate of y

↓ supports complex #s

for reals, $y^* = y$.