02/02

Guerilla section - Sunday 1-6p. @ 400 Cory

Review:

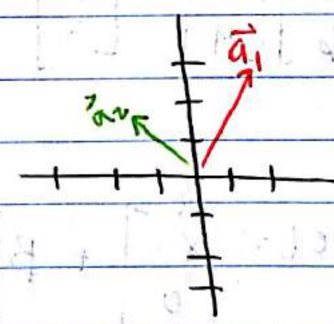
$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{\vec{b}}$$

$a_{ij} \to$ ith row, jth column

$$\begin{bmatrix} \underbrace{a_{11}x_1}_{x_1\vec{a}_1} + \underbrace{a_{12}x_1}_{x_2\vec{a}_2} \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

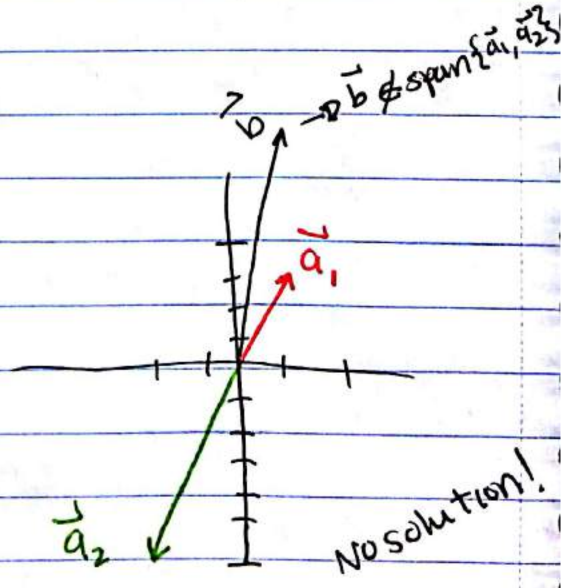
Linear combo of columns of A

Ex: $\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$



$$x_1 \overset{\vec{a}_1}{\begin{bmatrix} 1 \\ 3 \end{bmatrix}} + x_2 \overset{\vec{a}_2}{\begin{bmatrix} -2 \\ 2 \end{bmatrix}} = \overset{\vec{b}}{\begin{bmatrix} 1 \\ 11 \end{bmatrix}}$$

Unique sol: $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$



Ex: $\begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}$

$$x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}$$

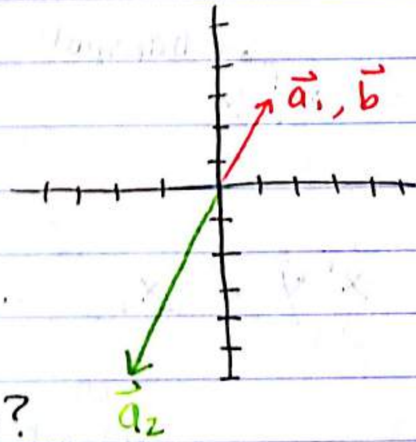
$2\vec{a}_1 + \vec{a}_2 = \vec{0} \Rightarrow$ linearly dependent.

span $\{\vec{a}_1, \vec{a}_2\}$ is the set of all points in $\mathbb{R}^n$ that is a linear combo of $\vec{a}_1$ & $\vec{a}_2$.

$= \{\vec{q} \in \mathbb{R}^n \mid \vec{q} = \alpha_1 \vec{a}_1 + \alpha_2 \vec{a}_2 \ \exists\ \alpha_1, \alpha_2 \in \mathbb{R}\}$

Ex

$$\begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$



$$\vec{b} = \vec{a}_1 = [\vec{a}_1, \vec{a}_2] \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$b \in \text{span}\{\vec{a}_1, \vec{a}_2\}$

$\longrightarrow$ Why infinite # of solutions?

$\vec{x}_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (particular sol)

Recall $2\vec{a}_1 + \vec{a}_2 = 0. \longrightarrow \begin{bmatrix} a_1 & a_2 \end{bmatrix} \underbrace{\begin{bmatrix} 2 \\ 1 \end{bmatrix}}_{\vec{x}_h \text{ (homogeneous sol)}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

General Solution $\vec{x} = \vec{x}_p + \underset{\in \mathbb{R}}{\lambda} \vec{x}_h$

$A\vec{x} = A(\vec{x}_p + \lambda \vec{x}_h) = \underbrace{A\vec{x}_p}_{\vec{b}!} + \underbrace{\cancel{A \lambda \vec{x}_h}}_{0!} = \vec{b}.$

More generally:

$A \in \mathbb{R}^{N \times N} \quad \vec{x} \in \mathbb{R}^N$

then, $A\vec{x} = x_1 \vec{a}_1 + \ldots + x_N \vec{a}_N = \sum_{n=1}^{N} x_n \vec{a}_n$

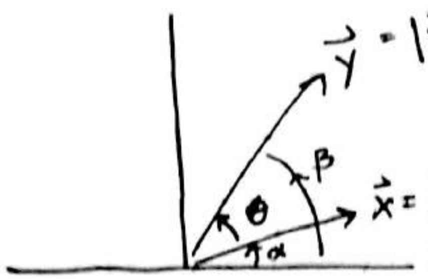
Dot Product:

$$\vec{x} \cdot \vec{x} = [x_1, \ldots, x_n] \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 x_1 + \ldots + x_n x_n$$

$$= \underbrace{||\vec{x}||^2}_{\text{2-Norm}}$$

$\vec{x} \cdot \vec{x} = x^T x$ ← transpose $\quad x \; N \times 1 \quad x^T \; 1 \times N$

$$\vec{x} \cdot \vec{y} = \vec{x}^T \vec{y} = [x_1, \ldots, x_n] \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \sum_{n=1}^{N} x_n y_n$$



$\vec{y} = |\vec{y}| \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix}$

$\vec{x} = \begin{bmatrix} |x|\cos\alpha \\ |x|\sin\alpha \end{bmatrix}$ $\quad \theta = \beta - \alpha.$

$$\vec{x} \cdot \vec{y} = |\vec{x}||\vec{y}| \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} \cdot \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix}$$

$$= |\vec{x}||\vec{y}| \underbrace{\cos(\alpha - \beta)}_{\cos\theta!}$$

If $\vec{x} \cdot \vec{y} = \vec{x}^T \vec{y} = 0 \rightarrow \vec{x} \perp \vec{y}$

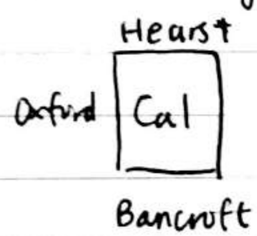
$$A\vec{x} = [\vec{a}_1 \quad \vec{a}_2] \begin{bmatrix} \vec{x}_1 \\ \vec{x}_2 \end{bmatrix} = x_1\vec{a}_1 + x_2\vec{a}_2$$

Row Interpretation

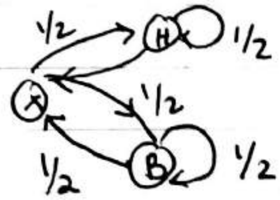
$$A = \begin{bmatrix} \vec{a}_1^T \\ \vec{a}_2^T \end{bmatrix} \qquad A\vec{x} = \begin{bmatrix} \vec{\alpha}_1^T \\ \vec{\alpha}_2^T \end{bmatrix} \vec{x} = \begin{bmatrix} \vec{\alpha}^T \vec{x} \\ \vec{\alpha}_2^T \vec{x} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \qquad \begin{array}{l} \vec{\alpha}_1^T = [1 \quad -2] \\ \vec{\alpha}_2^T = [3 \quad 2] \end{array} \qquad \begin{array}{l} A\vec{x}_n = \vec{0} \Rightarrow \\ \vec{x}_n \perp \vec{\alpha}_1^T, \vec{\alpha}_2^T. \end{array}$$

Matrices & Dynamic Systems



assume discrete time



n: current time

arrow represents next tick of clock

$$\vec{S}[n] = \begin{bmatrix} S[B] \\ S[X] \\ S[H] \end{bmatrix} \quad \text{state of system}$$

$$\vec{S}[n+1] = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \vec{S}[n] = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

state transition matrix

$\vec{S}[0]$

$\vec{S}[1] = A\vec{S}[0]$

$\vec{S}[2] = A\vec{S}[1] = AA\vec{S}[0] = A^2\vec{S}[0]$!! $S[n] = A^n\vec{S}[0]$

$$A^2 = \underbrace{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}}_{[\vec{a}_1, \vec{a}_2, \vec{a}_3]} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$\uparrow$

$\vec{a}_1$

$$A[\vec{a}_1, \vec{a}_2, \vec{a}_3] = [A\vec{a}_1 \quad A\vec{a}_2 \quad A\vec{a}_3]$$

$$= \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$