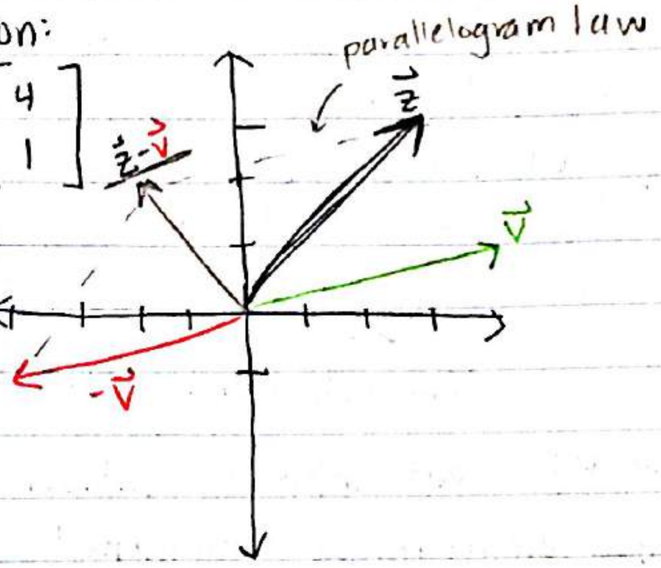
01/28/15

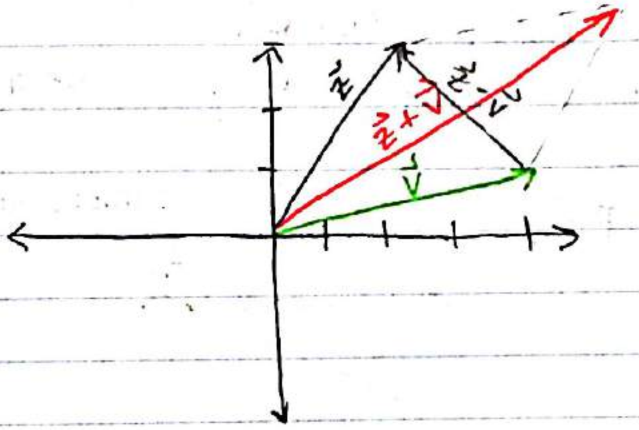
Vector Subtraction:

$\vec{z} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\vec{z} - \vec{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$



Notice:



$r = \alpha \vec{z} + \beta \vec{v}$ $\alpha, \beta \in \mathbb{R}$

$\alpha = 2, \beta = 1$

$r = 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$

α and β scale all the components of a vector.

Linear Combination of Vectors:

$\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m \in \mathbb{R}^N$ LinComb $= \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \ldots + \alpha_m \vec{v}_m$

$\alpha_1, \alpha_2, \ldots, \vec{\alpha}_m \in \mathbb{R}$

Matrix Vector Multiplication:

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2}$ a_{ij} = elem of ith row, jth column

$A\vec{x} = \vec{b}$, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$= \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}_{2\times1} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

*when we say $\vec{r} = \vec{s}$, $\vec{r}$ has same dim as $\vec{s}$ and

$\vec{r}_1 = \vec{s}_1$

$\vdots = \vdots$

$\vec{r}_k = \vec{s}_k$

Alternative Interpretation:

$A = \begin{bmatrix} \boxed{\begin{matrix} a_{11} \\ a_{21} \end{matrix}} & \boxed{\begin{matrix} a_{12} \\ a_{22} \end{matrix}} \end{bmatrix} = \begin{bmatrix} \vec{a}_1, & \vec{a}_2 \end{bmatrix}$

a_1 a_2 ↑ 1st col   ↑ 2nd col

i.e. $A\vec{x} = [a_1, a_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

so $A\vec{x} = \vec{b}$ can be:

$x_1 \underbrace{\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}}_{a_1} + x_2 \underbrace{\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix}}_{a_2} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

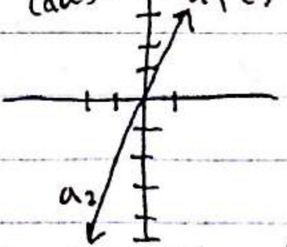
$= x_1\vec{a}_1 + x_2\vec{a}_2 = \vec{b}$

= linear combo of columns of A!

Ex. from prev. lecture:

$$\begin{matrix} x - 2y = 1 \\ 3x - 6y = 12 \end{matrix} \longrightarrow \underbrace{\begin{bmatrix} \overset{a_1}{1} & \overset{a_2}{-2} \\ 3 & -6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 1 \\ 12 \end{bmatrix}}_{\vec{b}}$$

no solution (doesnt lie on a_1 (1,12))



!can't apply parallelogram!

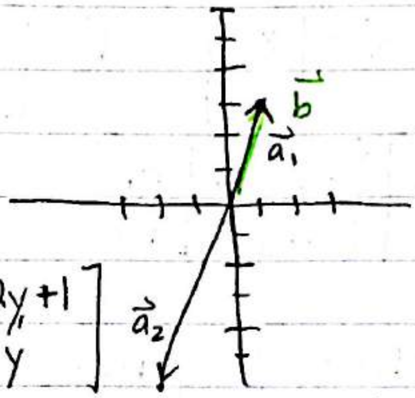
And we have $xa_1 + ya_2 = b$

$a_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $a_2 = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$ and $a_2 = -2a_1$

Ex. 2 of prev. lecture: (Infinite Sol)

$$\begin{matrix} x - 2y = 1 \\ 3x - 6y = 3 \end{matrix} \longrightarrow \begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$\vec{a}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$



One sol: $\vec{b} = \vec{a}_1$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 2y+1 \\ y \end{bmatrix}$$

infinite sol! $(y \in \mathbb{R})$

Linear Dependence & Independence of Vectors:

$\vec{a}_2 = -2\vec{a}_1 \longrightarrow \underbrace{2\vec{a}_1 + \vec{a}_2}_{\text{lin com. of cols of A}} = \vec{0}$.

At least one coeff in the linear combo of $\vec{a}_1$ & $\vec{a}_2$ is nonzero, linear combo produces $\vec{0}$ vector. Then, $\vec{a}_1$ and $\vec{a}_2$ are linearly dependent.

Def We say $\vec{v}_1, \ldots, \vec{v}_M \in \mathbb{R}^M$ are linearly dependent if there exists a set of coefficients $a_1, \ldots, a_M \in \mathbb{R}$ - not all of which are zero - such that $\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \ldots + \alpha_M \vec{v}_M = \vec{0}$.

Linear Independence: A set of vectors $v_1, \ldots, v_M$ is linearly independent if it is NOT linearly dependent

The only linear combination that produces $\vec{0}$ is $\alpha_1 = \alpha_2 = \ldots = \alpha_M = 0$

Q: can a linearly independent set of vectors include $\vec{0}$? $\vec{v}_1, \ldots, \vec{v}_M$

No! → $\alpha_1 \vec{0} + \alpha_2 \vec{v}_2 + \ldots + \vec{\alpha}_M \vec{v}_M = \vec{0}$.

$\uparrow = 0$

T/F: A set of vectors $\vec{v}_1, \ldots, \vec{v}_m \in \mathbb{R}^M$ is linearly dep if at least one of them can be expressed as a linear combo of the rest.

True: Linear dep. if combo = $\vec{0}$.

Since $\vec{v}_1 = \sum_{\ell=2}^{M} \beta_\ell \vec{v}_\ell$, $\vec{0} = \vec{v}_1 - \sum_{\ell=2}^{M} \beta_\ell \vec{v}_\ell$.

$$\beta_\ell = \frac{-\alpha_\ell}{\alpha_1}$$

Prev ex:

$$\begin{bmatrix} 1 & -2 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$a_2 = -2a_1$. → $2a_1 + a_2 = \vec{0}$.

$$\begin{bmatrix} a_1, a_2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

↳ gives you infinite sol.

Sol Set: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\beta \in \mathbb{R}$