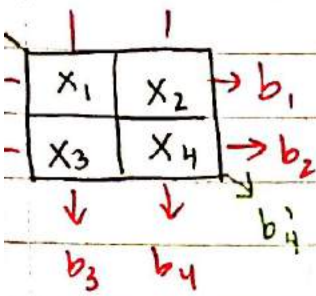
Babak OH: 12-2p Wed @ 531 Cory



$$
\begin{aligned}
x_1 + x_2 &= b_1 \\
x_3 + x_4 &= b_2 \\
x_1 + x_3 &= b_3 \\
x_2 + x_4 &= b_4
\end{aligned}
$$

→ E4 = E1 + E2 − E3, redundant.

BUT... in the real life, there is noise. Let's ignore for now.

Let's measure through x_1 and x_4.

We get: $\sqrt{2}(x_1 + x_4) = b_4' \rightarrow x_1 + x_4 = b_4/\sqrt{2}$

$= b_4'$ ✓

The 2 vertical & 2 horizontal measurements didn't work because $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ are indistinguishable

Matrix Notation:

$$
\begin{aligned}
x_1 + x_2 &= b_1 \\
x_3 + x_4 &= b_2 \\
x_1 + x_3 &= b_3 \\
x_1 + x_4 &= b_4
\end{aligned}
\Rightarrow
\begin{array}{cccc}
x_1 & x_2 & x_3 & x_4 \\
\end{array}
\left[\begin{array}{cccc|c}
1 & 1 & 0 & 0 & b_1 \\
0 & 0 & 1 & 1 & b_2 \\
1 & 0 & 1 & 0 & b_3 \\
1 & 0 & 0 & 1 & b_4
\end{array}\right]
$$

$\underbrace{\quad\quad\quad}_{A_{4\times 4}}$

unknowns: $\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}}_{\vec{b}}$ → This is an $A\vec{x} = \vec{b}$.

Solving Linear Eqns: Gaussian Elimination

e.g. I $\quad \begin{matrix} x-2y=1 \\ 3x+2y=11 \end{matrix} \rightarrow \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

$\downarrow$

II $\quad \begin{matrix} x-2y=1 \\ 8y=8 \end{matrix} \rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

upper-triangular form (y is isolated)

Procedure:

- Forward elimination: eliminate coefficient of x in eqn 2
- Isolate 2nd variable: Solve for it (y)
- Back substitution: Substitute y into E1 to get x.

1st pivot

$\begin{bmatrix} 1 & -2 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

2nd pivot

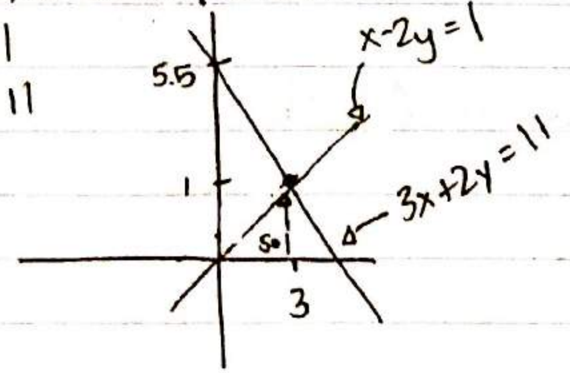
$y=1,$
$x-2=1$
$x=3.$

$\rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$

* use E1 to eliminate #s under E1's pivot.

Geometric Interpretation:

$x-2y=1$
$3x+2y=11$



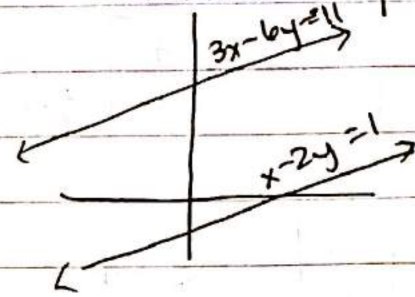
If a pivot is 0 → Stop!

Case I: Failure w/ no solution.

$x-2y=1$
$3x-6y=-11$

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & -6 & -11 \end{array}\right]$$

Geometric Interp:



$\rightarrow$ EQN 2 − 3 EQN 1

$$= \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & -14 \end{array}\right]$$

contradiction! $\therefore$ no solution

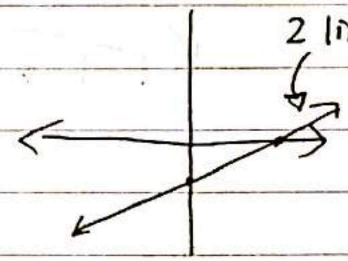
Case II : Failure, but infinite solutions!

$x-2y=1$ $\leftarrow$ doesn't reveal anything!
$3x-6y=3$

EQN 2 = 3 EQN 1

we get $0y=0$ $\leftarrow$ y can be anything.

Geometric Interp.:



Case III : Temp. Failure.

$0x + 2y = 4$
$3x - 2y = 5$ $\Rightarrow$ row exchange!

1st pivot.
$3x - 2y = 5$
$0x + 2y = 4$
$\rightarrow y = 2$
$\rightarrow x = 3$

Vectors:

$$\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \text{ or } \vec{v} = \begin{bmatrix} v_0 \\ \vdots \\ v_{n-1} \end{bmatrix}$$

$v_k \in \mathbb{R}$ $\leftarrow$ real numbers.

Vector Addition:

$$\vec{z} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \quad \vec{s} = \vec{v} + \vec{z} = \begin{bmatrix} 2+4 \\ 3+1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$